Lecture 6

2023/2024

Microwave Devices and Circuits for Radiocommunications

2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 16-18, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2023/2024

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 20.02.2024)
    - personal homework

Materials

- http://rf-opto.etti.tuiasi.ro

Site

English | Romana |

Main Courses Master Staff Research Students

Microwave and Optoelectronics Laboratory

We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (**ETTI**) from the "Gh. Asachi" Technical University (**TUIASI**) in Iasi, Romania

We currently cover inside **ETTI** the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

Courses

Nr.	Course	Shortcut	Code	Type	Semester	Credits	Weekly	Examination	Link
1	Microwave Devices and Circuits for Radiocommunications	**DCMR**	DOS412T	DOS	7	4	0P,1L,0S,2C	Exam	details
2	Monolithic Microwave Integrated Circuits	**CIMM**	RD.IA.207	DOMS	11	6	1.5L,0S,2C,0P	Exam	details
3	Advanced Techniques in the Design of the Radio-communications Systems	**TAPSR**	RD.IA.103	DIMS	9	6	1.5P,0L,0S,2C	Exam	details
4	Optical Communications	**CO**	DOS409T	DOS	7	5	0P,1L,0S,3C	Colloquium	details
5	Optical Communications	**OC**	EDOS409T	DOS	7	5	0P,1L,0S,3C	Exam	details
6	Satellite Communications	**CS**	RC.IA.104	DIMS	9	6	0L,0S,2C,1.5P	Exam	details
7	Applied Informatics 1	**IA1**	DOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
8	Applied Informatics 1	**AI1**	EDOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
9	Databases, Web Programming and Interfacing	**DWPI**	ITT.IA.601	DIS	11	5	1P,1L,0.25S,1C	Verification	details
10	Web Applications Design	**PAW**	RC.IA.108	DIMS	10	5	1L,0S,1.5C,1P	Exam	details
11	Optoelectronics	**OPTO**	DID405M	DID	8	4	0P,1L,0S,2C	Colloquium	details
12	Microwave Devices and Circuits for Radiocommunications (English)	**MDCR**	EDOS412T	DOS	8	4	0P,1L,0S,2C	Exam	details

Site

- New online exams
  - Supplemental points for lectures 3, 4, 5

Disciplina: MDCR (Microwave Devices and Circuits for Radiocommunications (Engleza))

Pas 3

Nr.	Titlu	Start	Stop	Text	Subiecte
1	Profile photos	05/03/2024; 08:00	01/06/2024; 08:00	Online "exam" created f ...	fotografii_en.pdf
2	Laboratory 2	19/03/2024; 08:00	10/04/2024; 18:00	Individual subjects for ...	Subjects_lab2_2024.pdf
3	Lecture 5 Network Analysis - supplemental points	19/03/2024; 14:00	10/04/2024; 11:00	Supplemental points for ...	Supliment_c5_2024.pdf
4	Lecture 3 Impedance Matching - supplemental points	19/03/2024; 08:00	03/04/2024; 11:00	Supplemental points for ...	Supliment_c3_2024.pdf
5	Lecture 4 Impedance Transformers - supplemental points	19/03/2024; 08:00	03/04/2024; 11:00	Supplemental points for ...	Supliment_c4_2024.pdf
6	Laboratory 1	05/03/2024; 08:00	27/03/2024; 14:00	Individual subjects for ...	Subjects_lab1_2024.pdf

Materials

- RF-OPTO
  - http://rf-opto.etti.tuiasi.ro
- David Pozar, "Microwave Engineering", Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/**online exam > Week4-Week6**
  - used at lectures/laboratory

Online – Registration no.

- access to **online exams** requires the **password** received by email

Password

- received by email

Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply | Reply all | Forward

Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020	[redacted]
[redacted]	[redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

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Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~

Materials

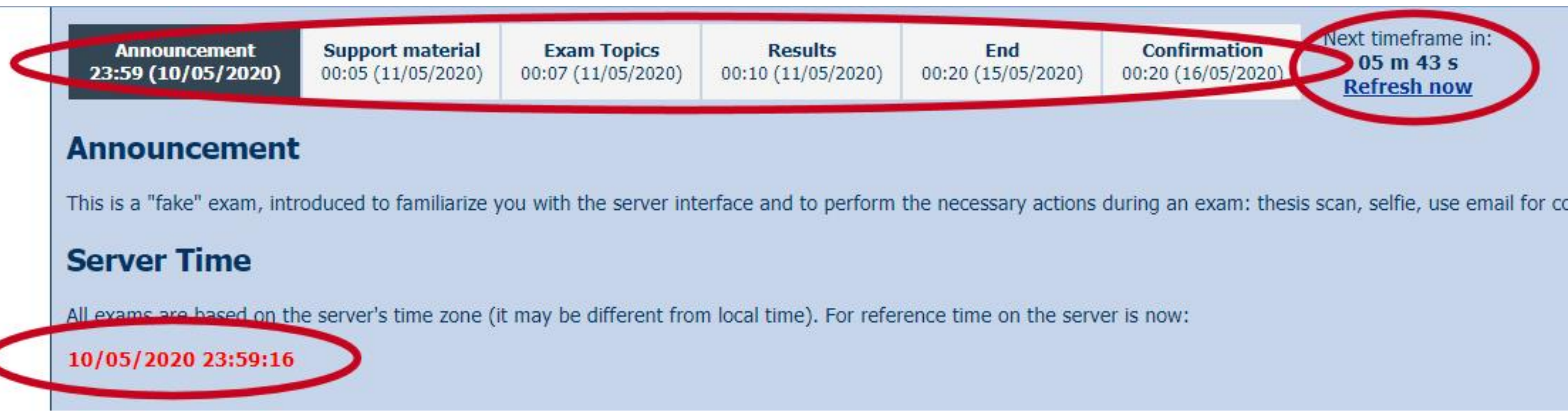
Other data

Manual examen on-line (pdf, 2.65 MB, ro,)
Simulare Examen (video) (mp4, 65.12 MB, ro,)

Microwave Devices and Circuits (Englis

Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri s	T2, fisier parmetri s	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

Grade = Quality of the work +

+ Quality of the submission

General theory

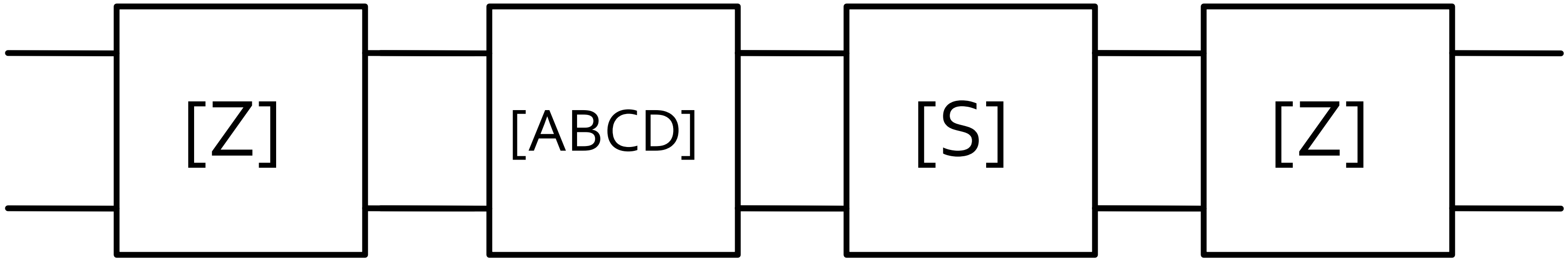
Microwave Network Analysis

Course Topics

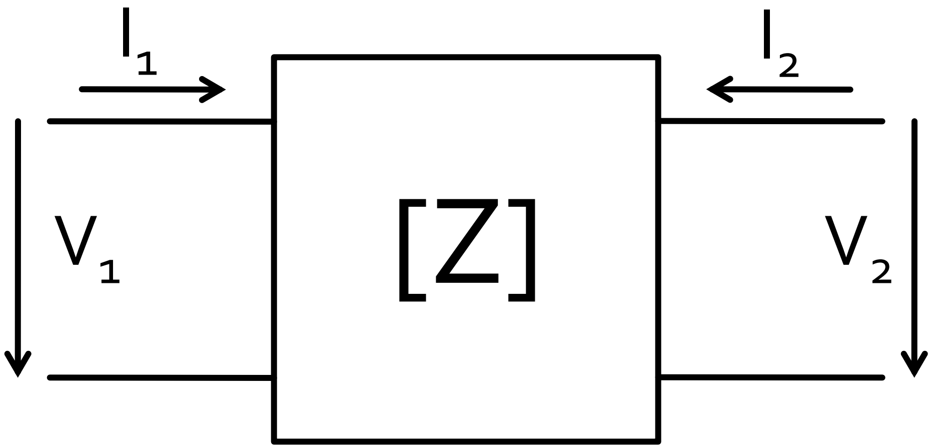
- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Network Analysis

- We try to separate a complex circuit into individual blocks
- These are analyzed separately (decoupled from the rest of the circuit) and are characterized only by the port level signals (**black box**)
- Network-level analysis allows you to put together individual block results and get a total result for the entire circuit



Impedance matrix – Z



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

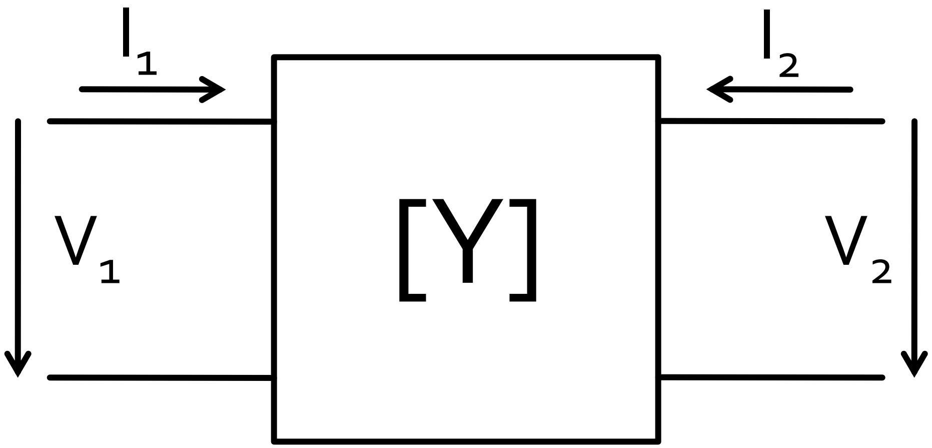
$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$V_1 = Z_{11} \cdot I_1 \Big|_{I_2=0} \qquad Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

- Z11 – input impedance with open-circuited output

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

Admittance matrix – Y



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

$$I_1 = Y_{11} \cdot V_1 \Big|_{V_2=0} \qquad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

- Y11 – input admittance with short-circuited output

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

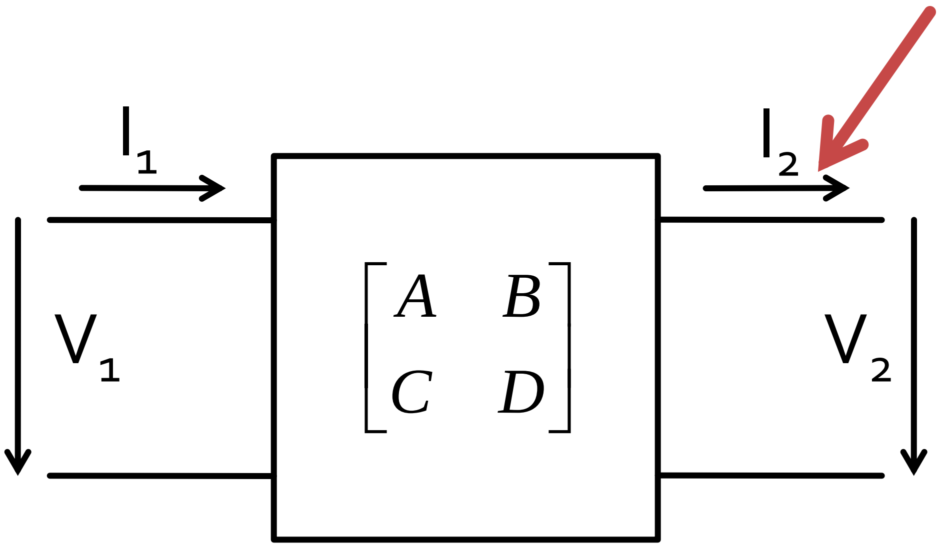
Network Analysis

- Each matrix is best suited for a particular mode of port excitation (V, I)
  - matrix H in common emitter connection for TB: I_B, V_{CE}
  - matrices provide the associated quantities depending on the “attack” ones
- Traditional notation of Z, Y, G, H parameters is in lowercase (z, y, g, h)
- In microwave analysis we prefer the notation in uppercase to avoid confusion with the **normalized parameters**

$$z = \frac{Z}{Z_0} \qquad y = \frac{Y}{Y_0} = \frac{1/Z}{1/Z_0} = \frac{Z_0}{Z} = Z_0 \cdot Y$$

$$z_{11} = \frac{Z_{11}}{Z_0} \qquad y_{11} = \frac{Y_{11}}{Y_0} = Z_0 \cdot Y_{11}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = A \cdot V_2 + B \cdot I_2$$

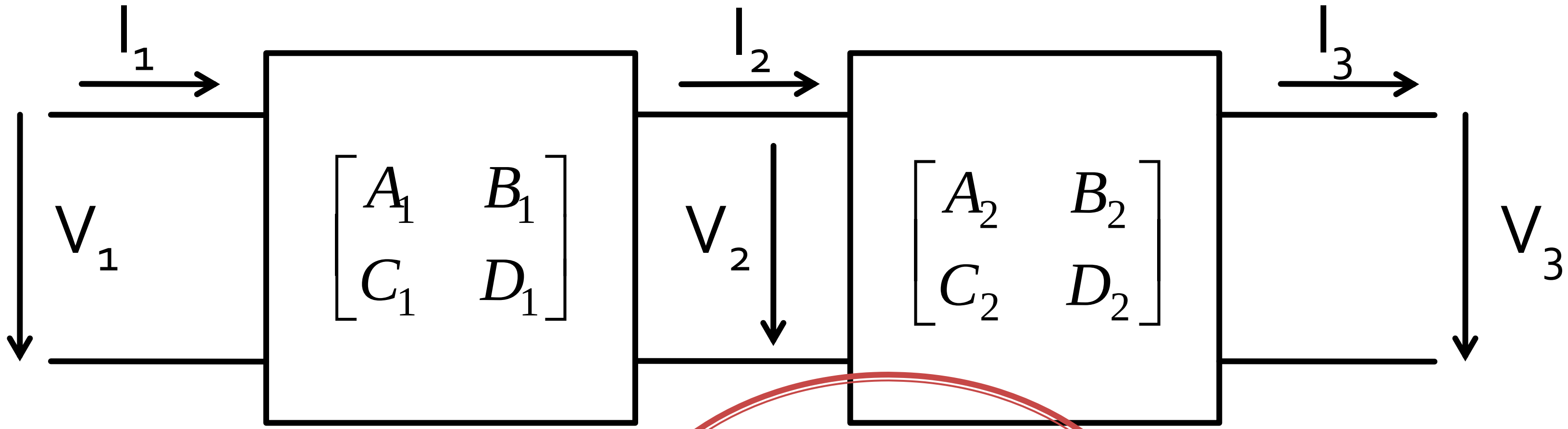
$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0} \qquad B = \left. \frac{V_1}{I_2} \right|_{V_2 = 0} \qquad C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0} \qquad D = \left. \frac{I_1}{I_2} \right|_{V_2 = 0}$$

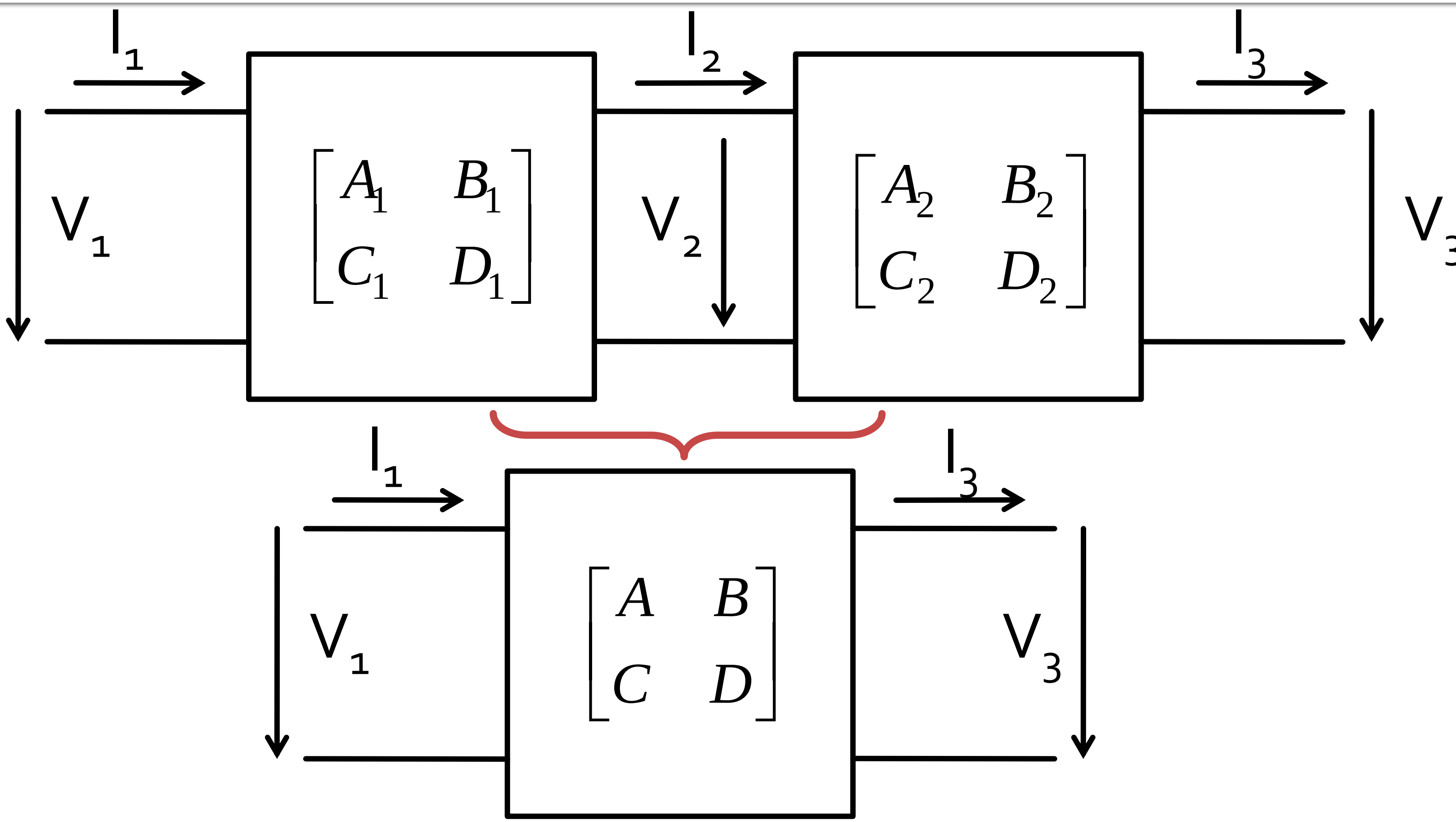
ABCD (transmission) matrix

- This 2X2 matrix characterizes the "input"/"output" relation
- Allows easy chaining of multiple two-ports



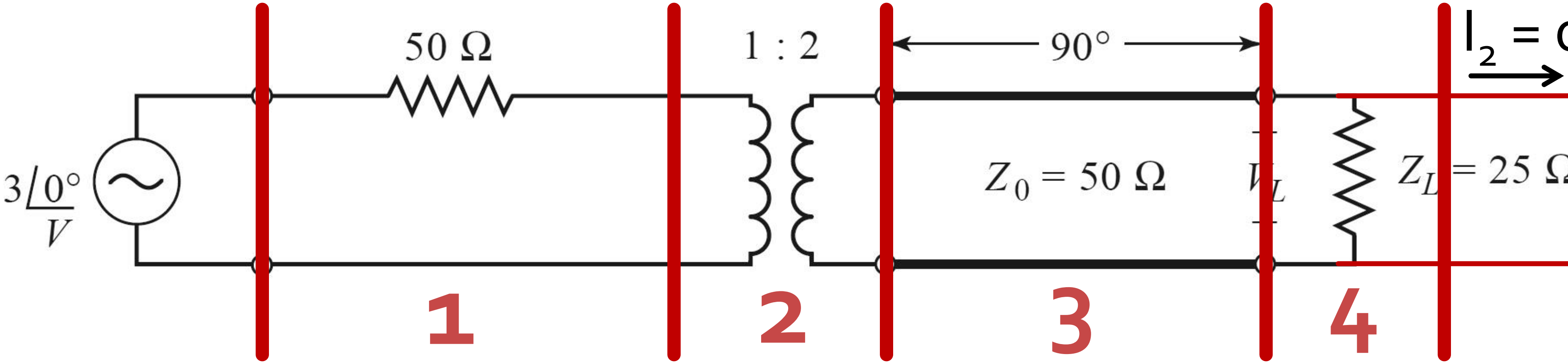
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

ABCD (transmission) matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Example for ABCD matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot j & 25 \cdot j \\ \frac{j}{25} & 0 \end{bmatrix}$$

$$V_L = \frac{V}{A} = \frac{3\angle 0°}{3 \cdot j} = 1\angle -90°$$

(Somewhat!) Specific theory

Microwave Network Analysis

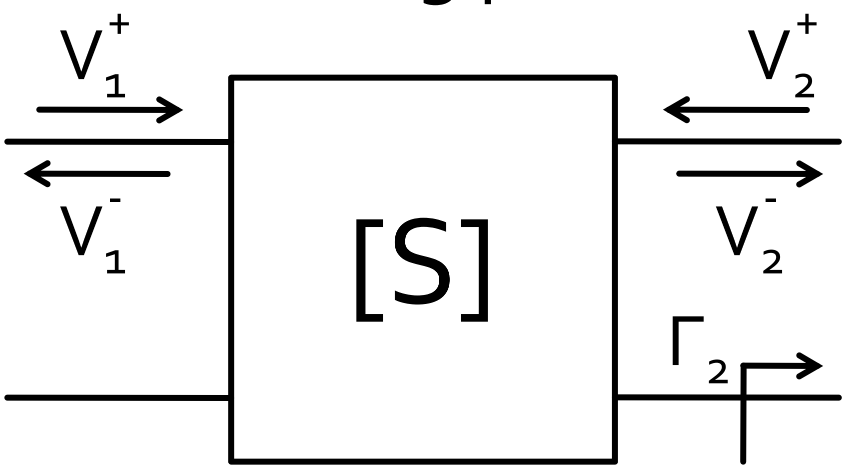
The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

- Average power flow is constant along the line
  - (**no** P_{avg}(**z**))
  - can be measured
- We can use the power to characterize the amplitude of a signal
  - a very "energetic" (basic physics) point of view
  - more power = "more" signal

Scattering matrix – S

- Scattering parameters



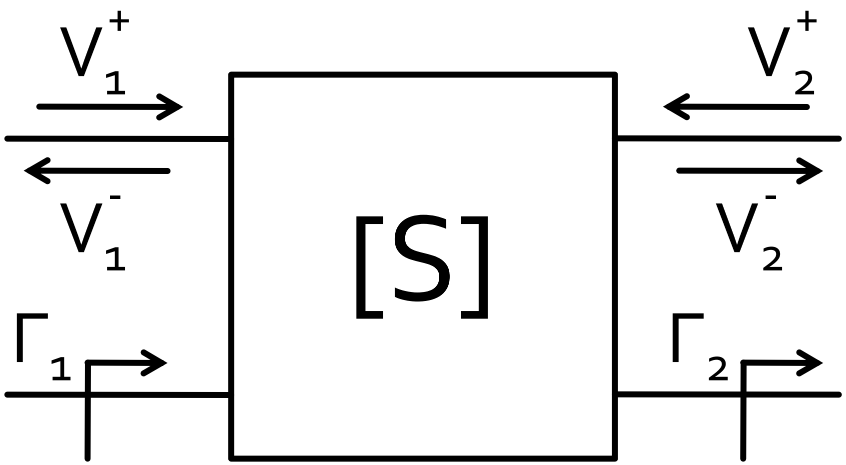
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \Gamma_1 \big|_{\Gamma_2=0}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = T_{21} \big|_{\Gamma_2=0}$$

- S11 is the reflection coefficient seen looking into port ***1*** when port ***2*** is terminated in matched load
- S21 is the transmission coefficient from port ***1*** (**second** index**!**) to port ***2*** (**first** index**!**) when port ***2*** is terminated in matched load

Generalized Scattering Parameters

- We define the power wave amplitudes a and b

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}}$$ the incident power wave

$$Z_R = R_R + j \cdot X_R$$

Any complex impedance, named reference impedance

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}}$$ the reflected power wave

- Total voltage and current in terms of the power wave amplitudes

$$V = \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}}$$

$$I = \frac{a - b}{\sqrt{R_R}}$$

Power waves

- When the load is conjugately matched to the generator

$$Z_g = Z_L^* \qquad P_{L\max} = \frac{1}{2}\cdot|a|^2 = \frac{V_0^2}{8\cdot R_L}$$

- Power reflection: L4

$$Z_L = Z_i^* \qquad P_{L\max} \equiv P_a \qquad \Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$

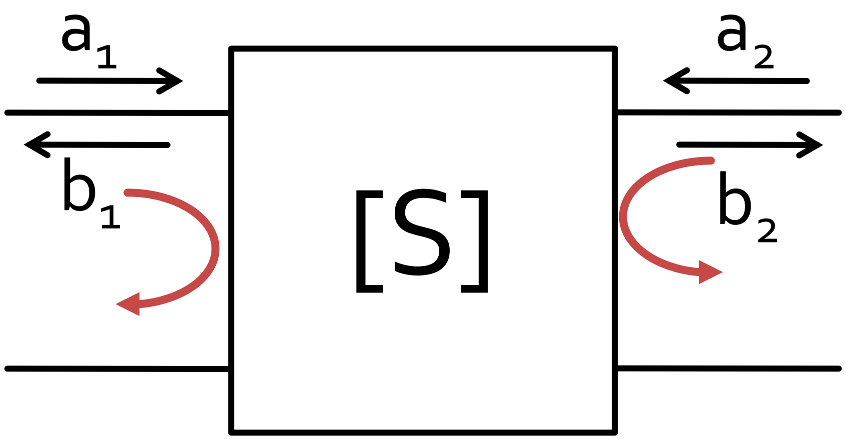
$$Z_L \neq Z_i^* \qquad P_r = P_a\cdot|\Gamma|^2 \qquad P_L = P_a - P_r = P_a - P_a\cdot|\Gamma|^2 = P_a\cdot\left(1-|\Gamma|^2\right)$$

- Power reflection: L5

$$P_{L\max} \equiv P_a = \frac{1}{2}\cdot|a|^2 \qquad P_L = \frac{1}{2}\cdot|a|^2 - \frac{1}{2}\cdot|b|^2 \qquad \Gamma_p = \frac{b}{a} = \frac{V - Z_R^*\cdot I}{V + Z_R\cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

$$P_L = \frac{1}{2}\cdot|a|^2 - \frac{1}{2}\cdot|a|^2\cdot|\Gamma_p|^2 \qquad P_L = P_a\cdot\left(1-|\Gamma_p|^2\right) \qquad P_r = P_a\cdot|\Gamma_p|^2 = \frac{1}{2}\cdot|b|^2$$

Scattering matrix – S

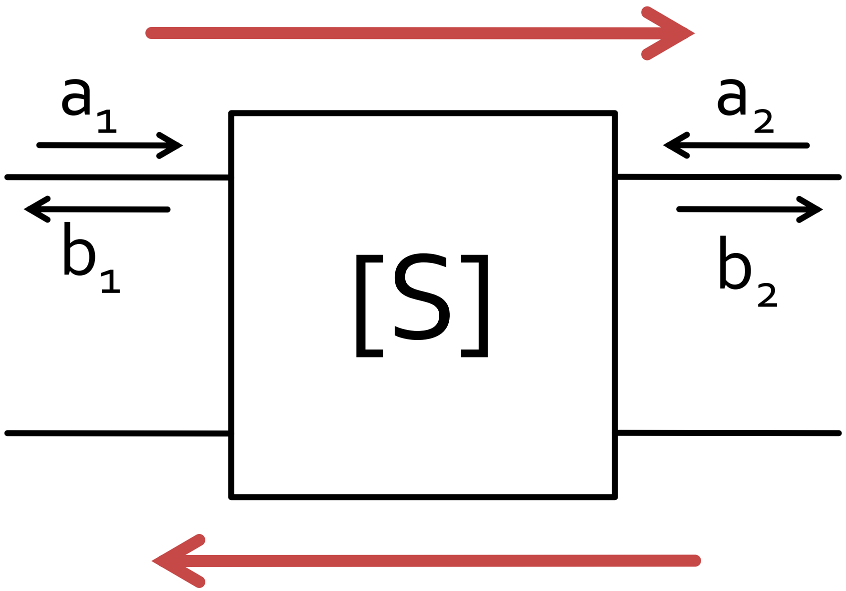


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \qquad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- S_{11} and S_{22} are reflection coefficients at ports 1 and 2 when the other port is matched

Scattering matrix – S

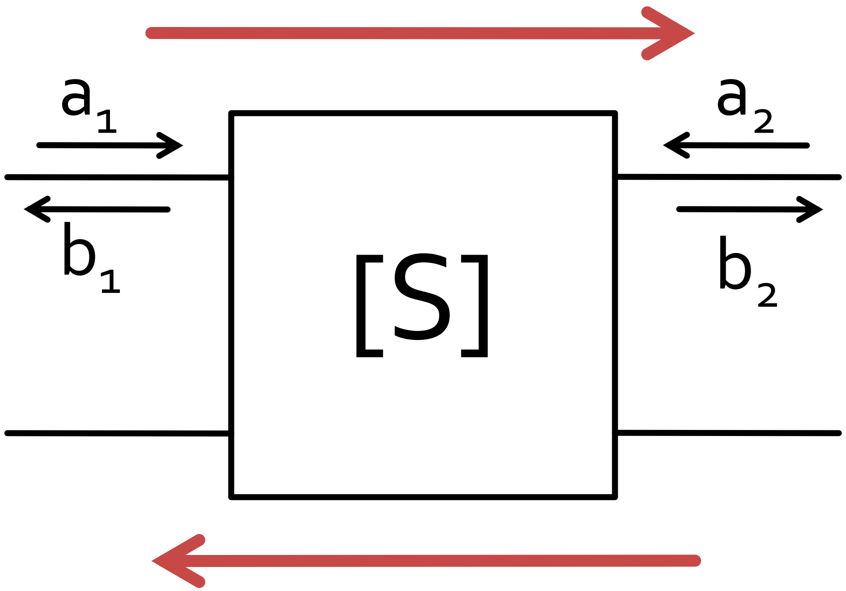


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude gain when the other port is matched

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Measuring S parameters - VNA

- Vector Network Analyzer

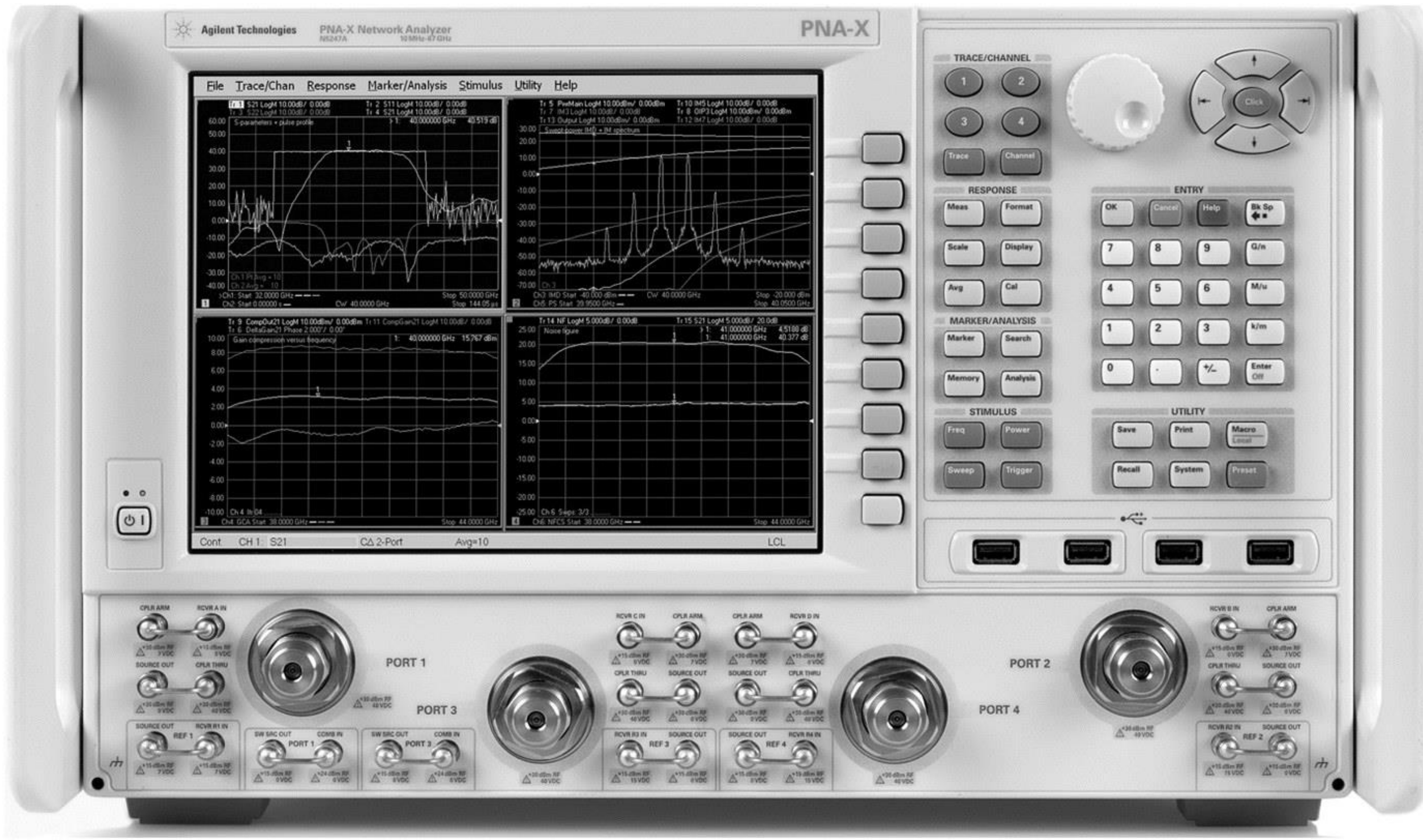
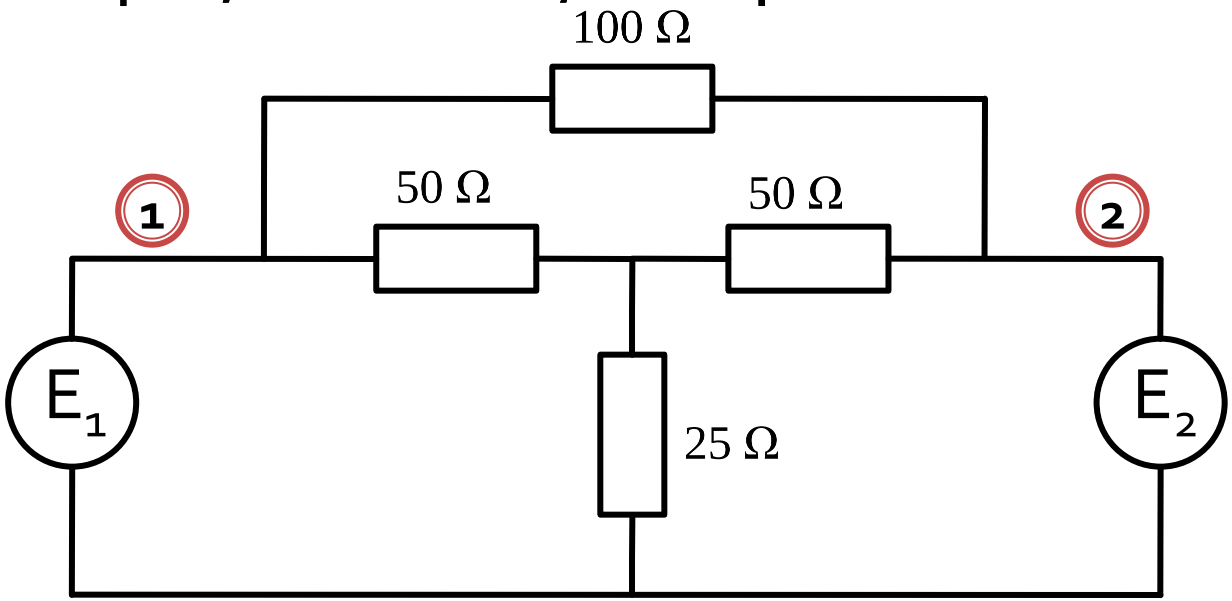


Figure 4.7
Courtesy of Agilent Technologies

Even/Odd Mode Analysis

Even/Odd Mode Analysis

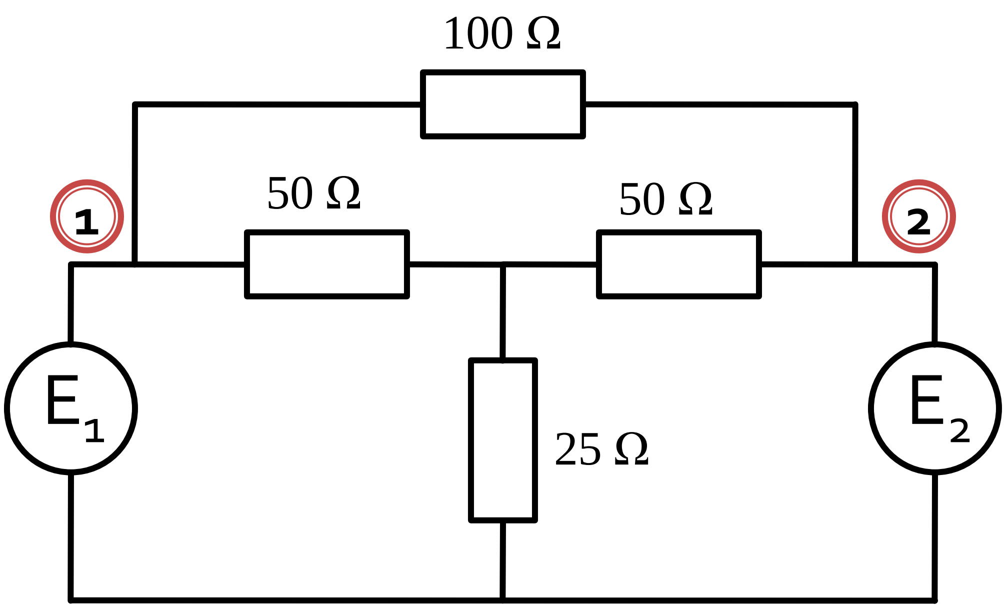
- useful method, necessary even for multiple ports
- example, resistors, two port circuit

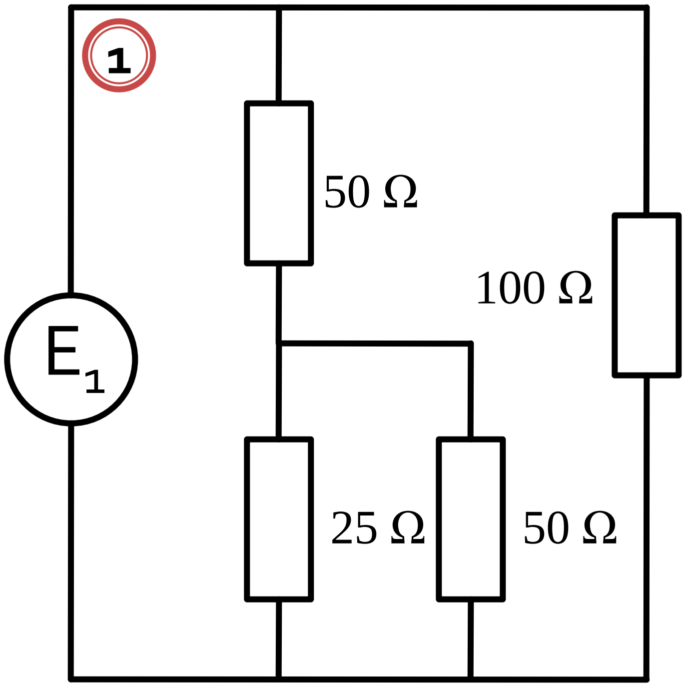


Even/Odd Mode Analysis

- assume we want to compute Y_{11}
- $E_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$





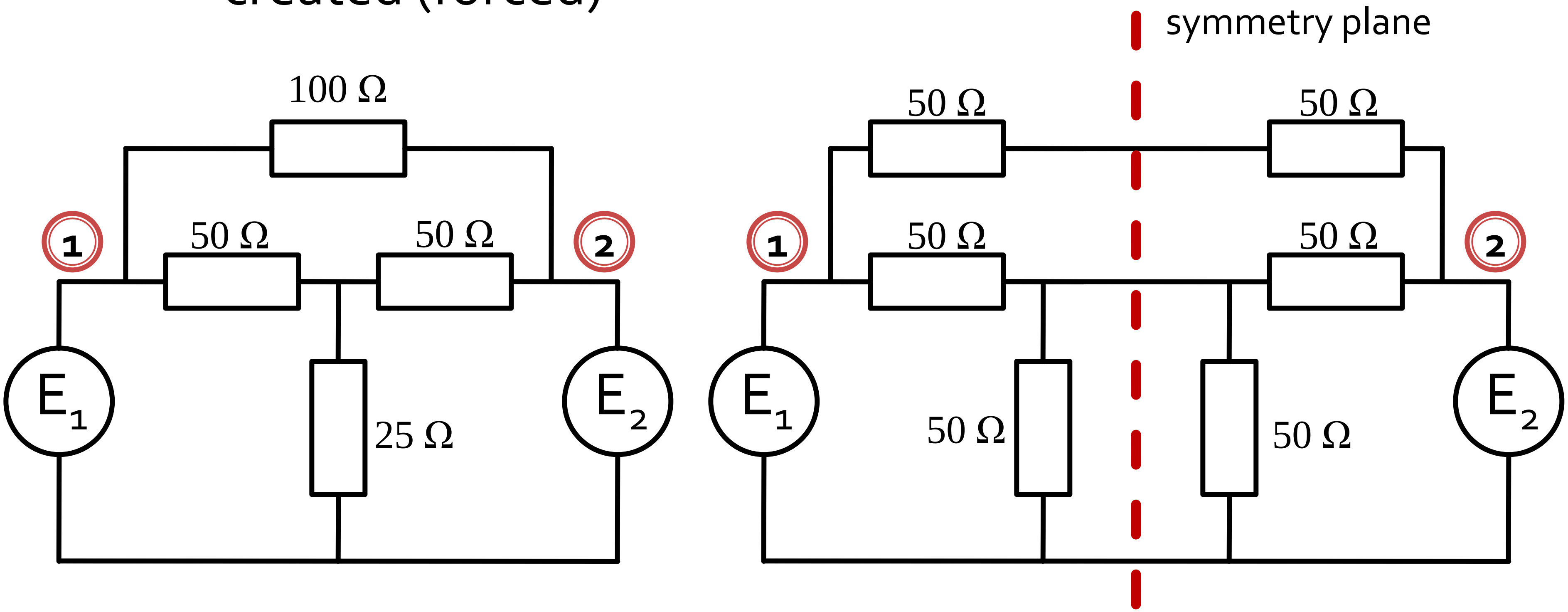
$$R_{ech} = 100\Omega \,||\, (50\Omega + 25\Omega \,||\, 50\Omega) =$$

$$= 100\Omega \,||\, (50\Omega + 16.67\Omega) = 100\Omega \,||\, 66.67\Omega = 40\Omega$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.025S$$

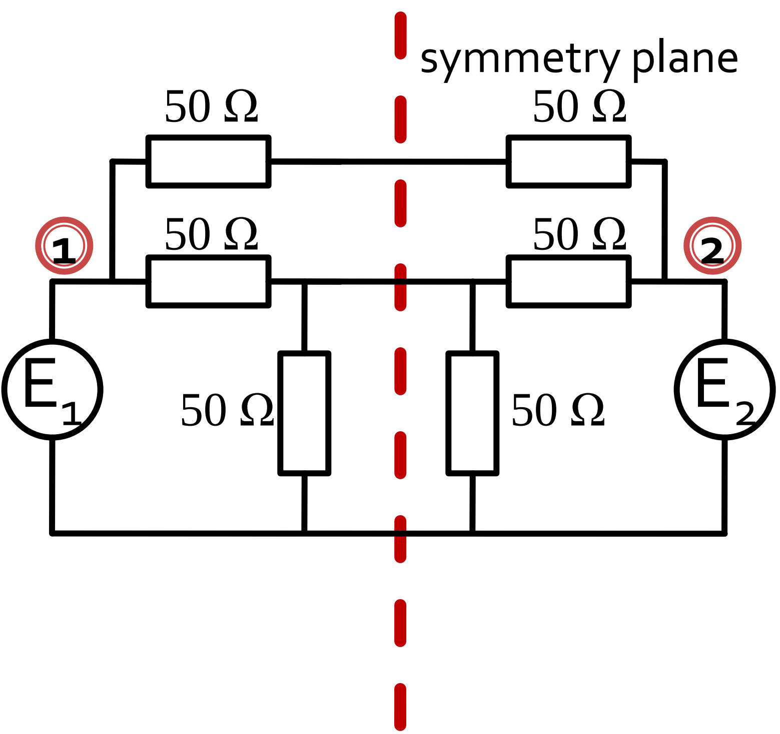
Even/Odd Mode Analysis

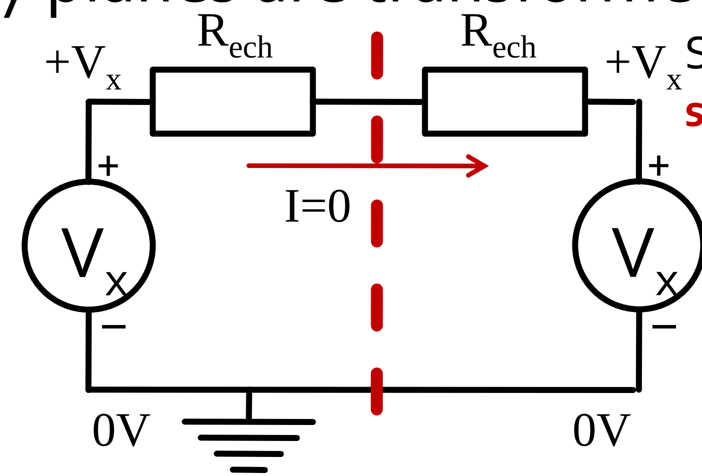
- Even/Odd mode analysis benefit from the existence of symmetry planes in the circuit
  - existing or
  - created (forced)



Even/Odd Mode Analysis

- when exciting the ports with symmetric/anti-symmetric sources the symmetry planes are transformed into:
  - open circuit
  - virtual ground

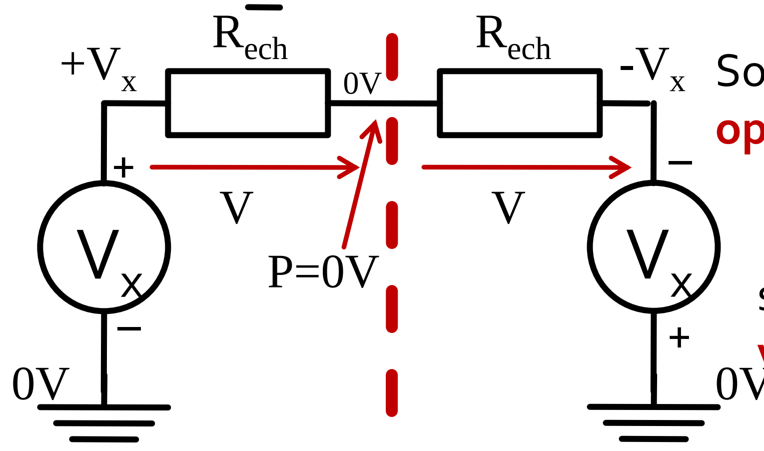




Sources have the **same** polarity

$$I = 0, \forall V_x$$

symmetry plane **open circuit**



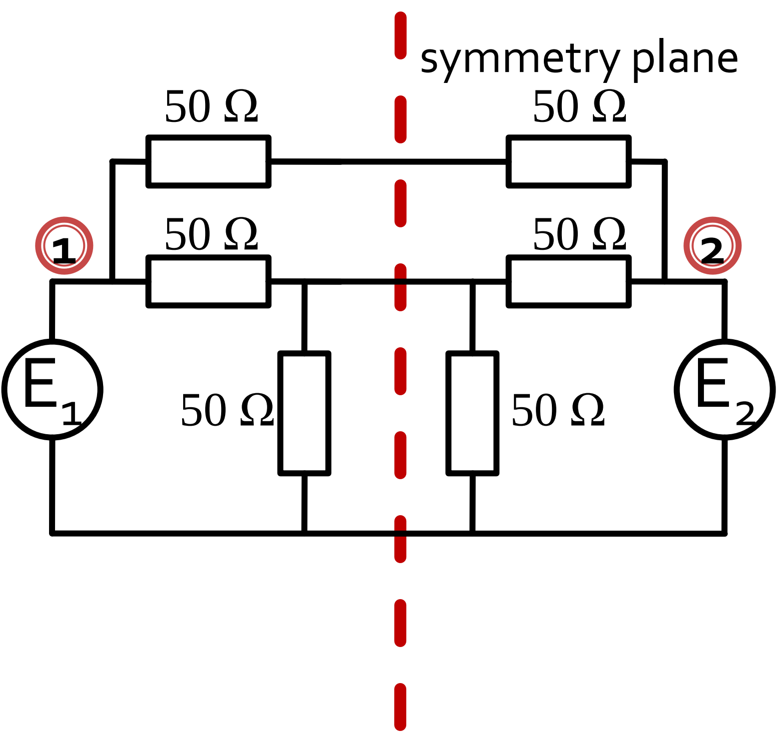
Sources have **opposite** polarities

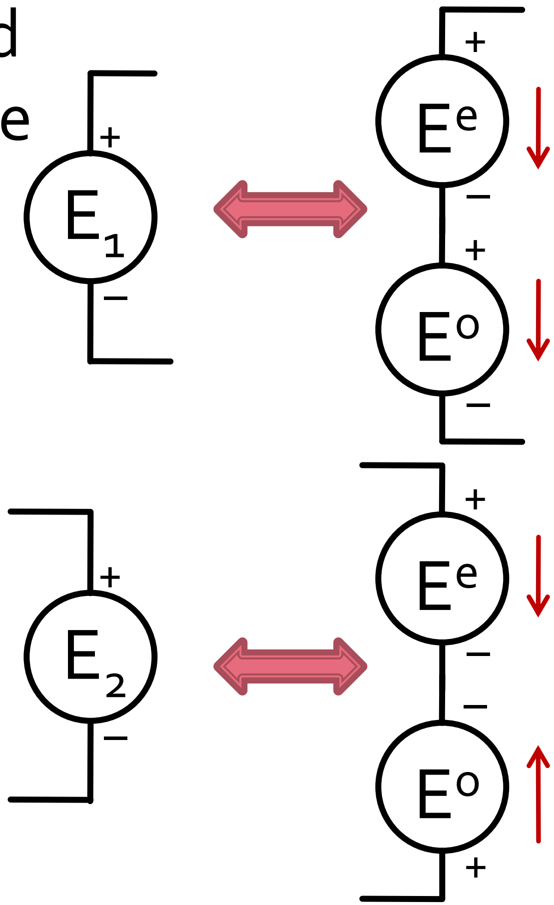
$$P = 0, \forall V_x$$

symmetry plane **virtual ground**

Even/Odd Mode Analysis

- the combination of any two sources is equivalent for linear circuits with the superposition of:
  - a symmetric source and
  - a anti-symmetric source





$$E_1 = E^e + E^o$$

$$E_2 = E^e - E^o$$

$$E^e = \frac{E_1 + E_2}{2}$$

$$E^o = \frac{E_1 - E_2}{2}$$

Even/Odd Mode Analysis

- In linear circuits the **superposition principle** is always true
  - the response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually

Response (**Source1** + **Source2**) =

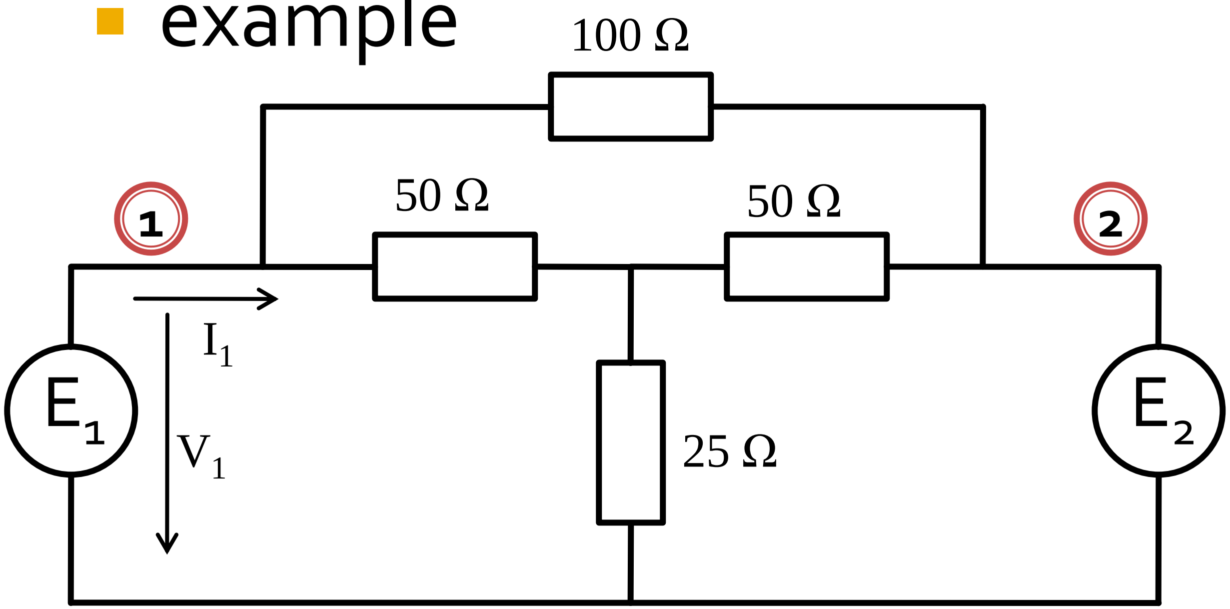
= **Response** (**Source1**) + **Response** (**Source2**)

Response(**ODD** + **EVEN**) = **Response** (**ODD**) + **Response** (**EVEN**)

We can benefit from existing symmetries !!

Even/Odd Mode Analysis

- example

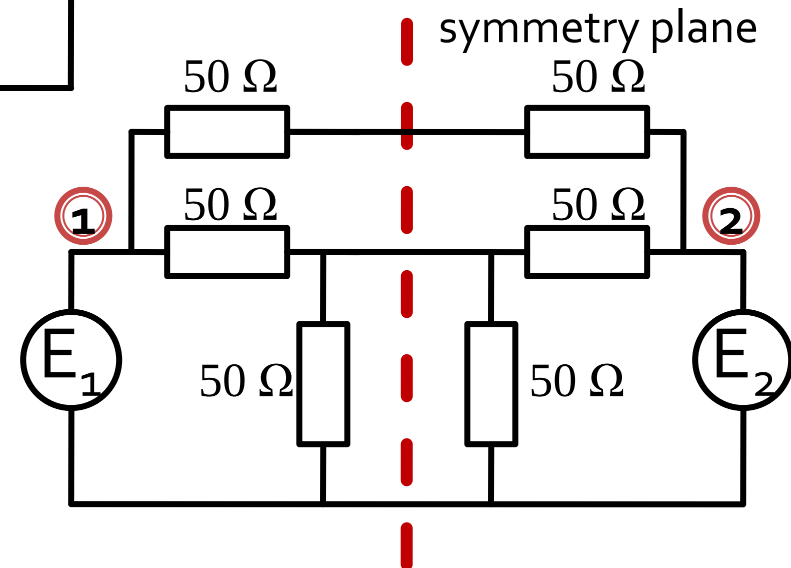


$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$V_2 \equiv E_2 = 0 \Rightarrow$$

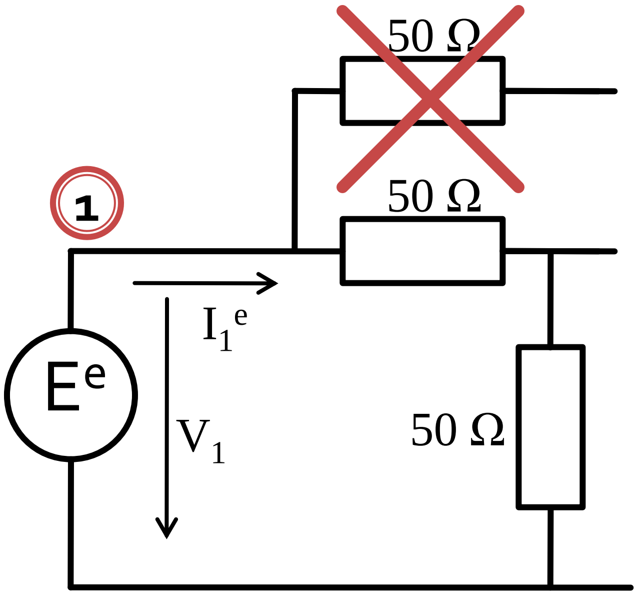
$$E^e = \frac{E_1}{2}$$

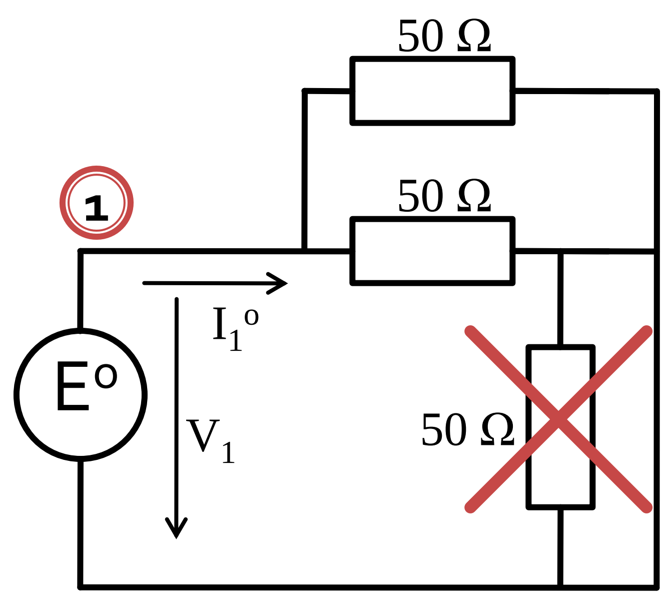
$$E^o = \frac{E_1}{2}$$



Even/Odd Mode Analysis

- Even/Odd mode analysis





$$R^e_{ech} = 50\Omega + 50\Omega = 100\Omega$$

$$I_1^e = \frac{E^e}{R^e_{ech}} = \frac{E_1/2}{100\Omega} = \frac{E_1}{200\Omega}$$

EVEN → symmetry plane **open circuit**

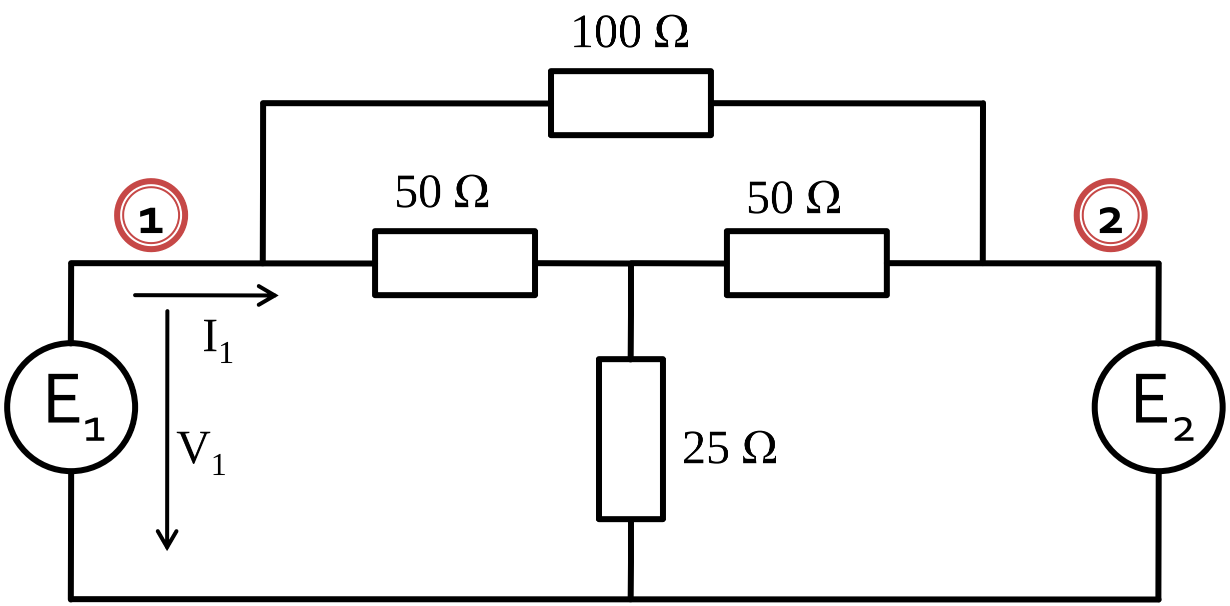
$$R^o_{ech} = 50\Omega \,||\, 50\Omega = 25\Omega$$

$$I_1^o = \frac{E^o}{R^o_{ech}} = \frac{E_1/2}{25\Omega} = \frac{E_1}{50\Omega}$$

ODD → symmetry plane **virtual ground**

Even/Odd Mode Analysis

- superposition principle



$$I_1 = I_1^e + I_1^o$$

$$V_1 = V_1^e + V_1^o$$

$$I_1 = I_1^e + I_1^o = \frac{E_1}{200\Omega} + \frac{E_1}{50\Omega} = \frac{E_1}{40\Omega}$$

$$V_1 = V_1^e + V_1^o = E_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{40\Omega} = 0.025S$$

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the **number of ports** (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Power dividers and directional couplers

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~

Introduction

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

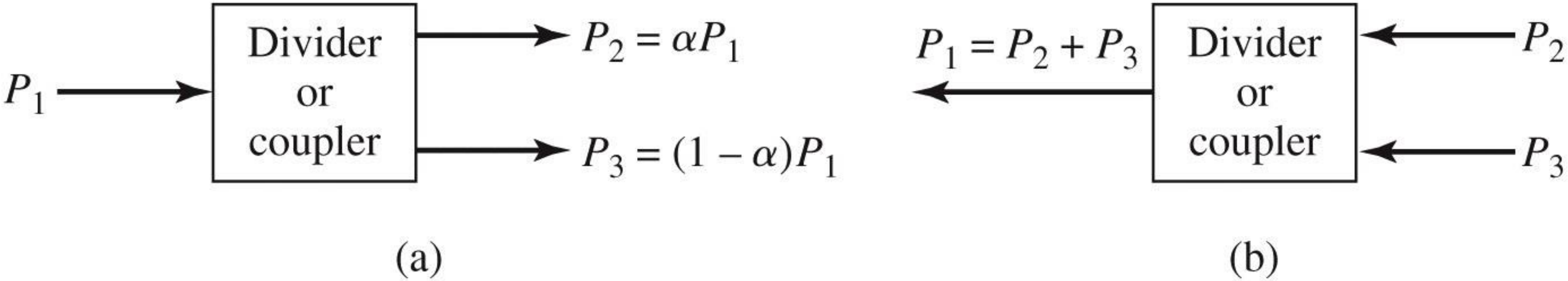
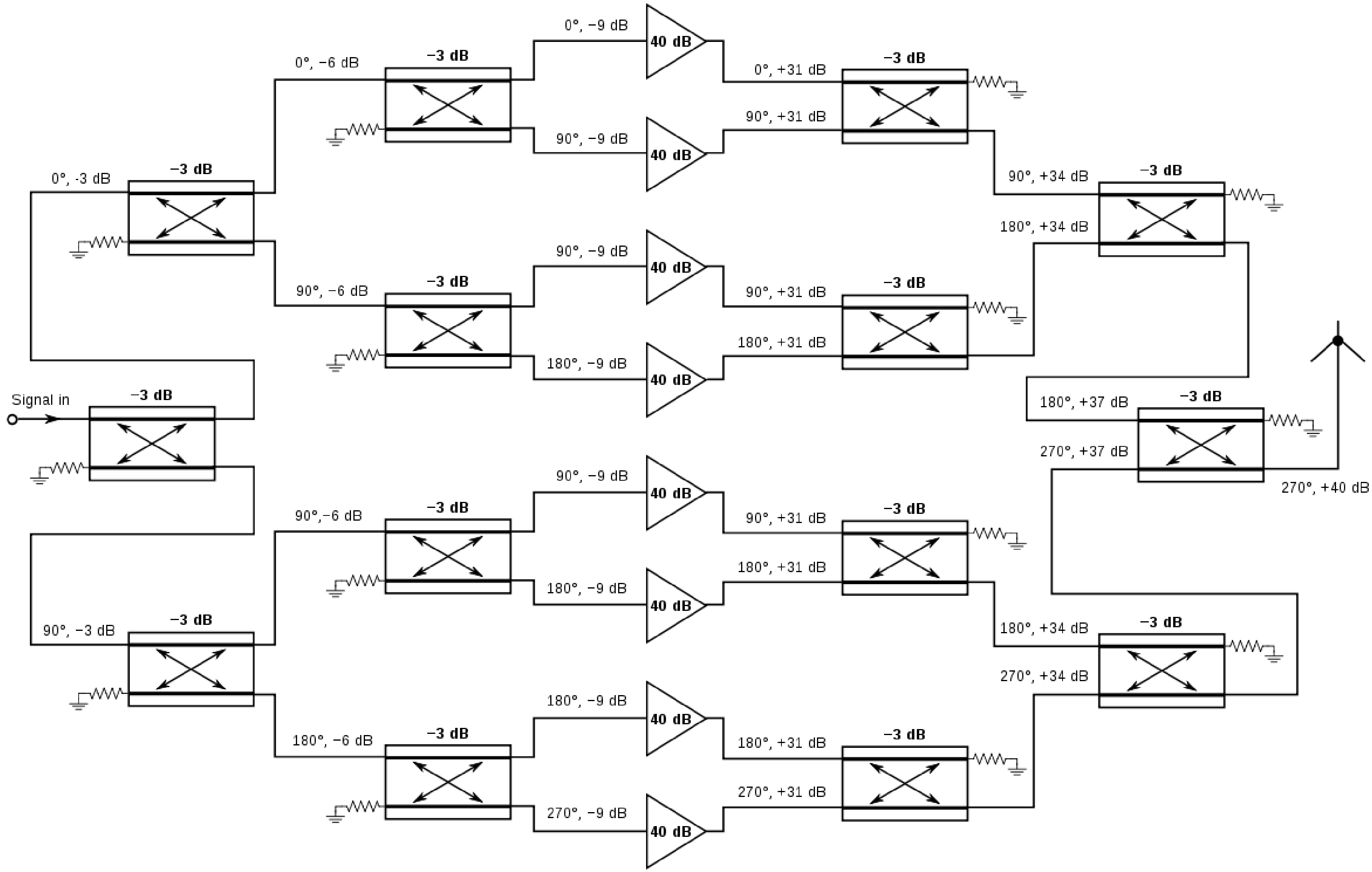


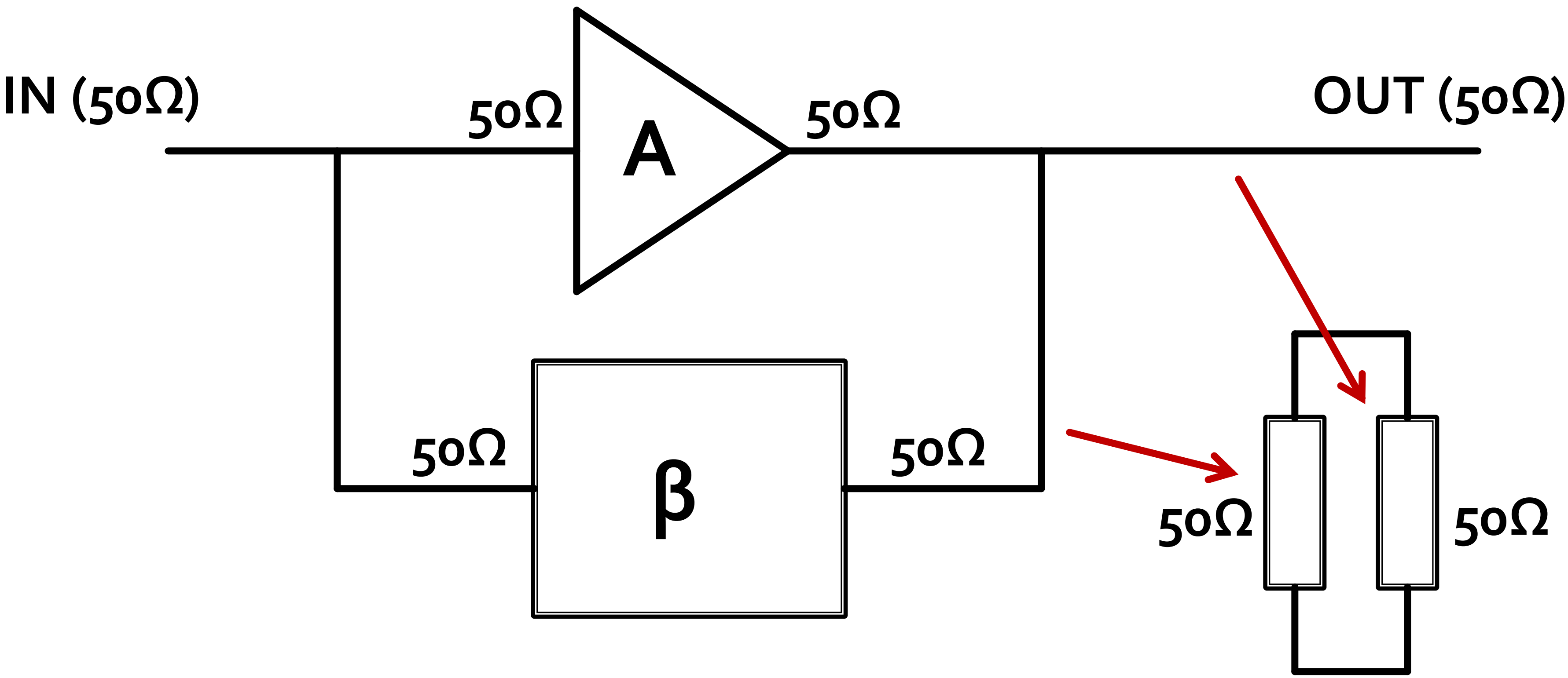
Figure 7.1

Balanced amplifiers



Matching

- feedback amplifier



Three-Port Networks

- also known as T-Junctions
- characterized by a 3x3 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power “loss”

Three-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Three-Port Networks

- lossless network

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^{*} = 1$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = 0, \forall i \neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{13}^{*} S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^{*} S_{13} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{23}^{*} S_{12} = 0$$

- no solution is possible

Three-Port Networks

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- **nonreciprocal**, but matched at all ports and lossless $S_{ij} \neq S_{ji}$
- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 6 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

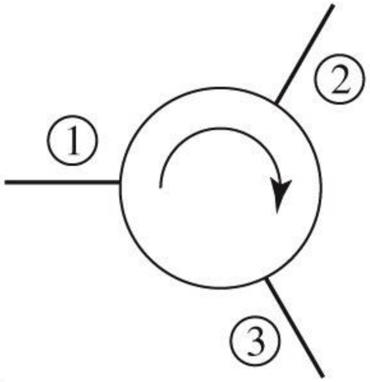
Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
  - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



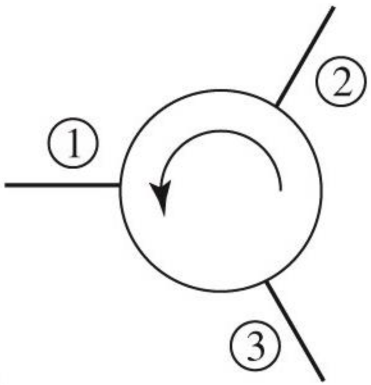
(a)

  - counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

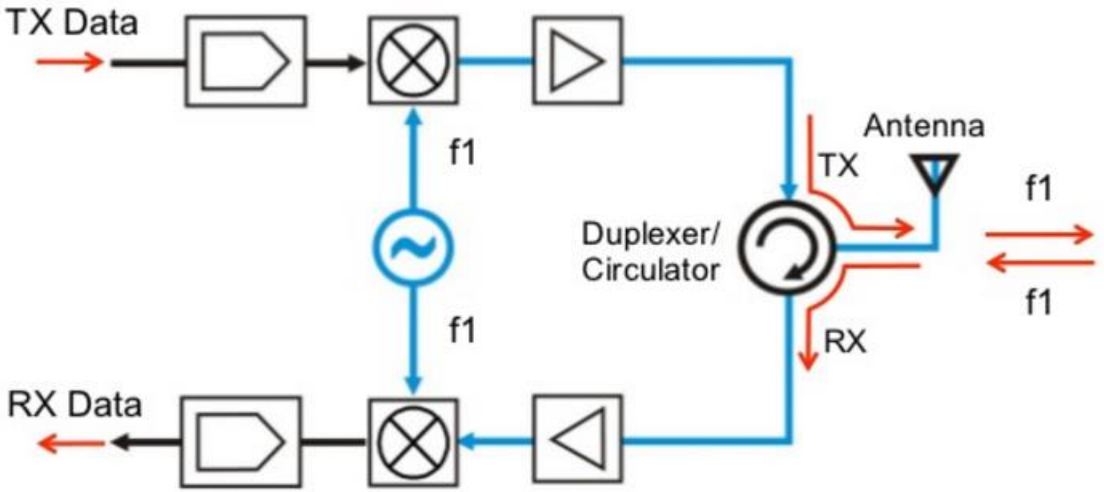
$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

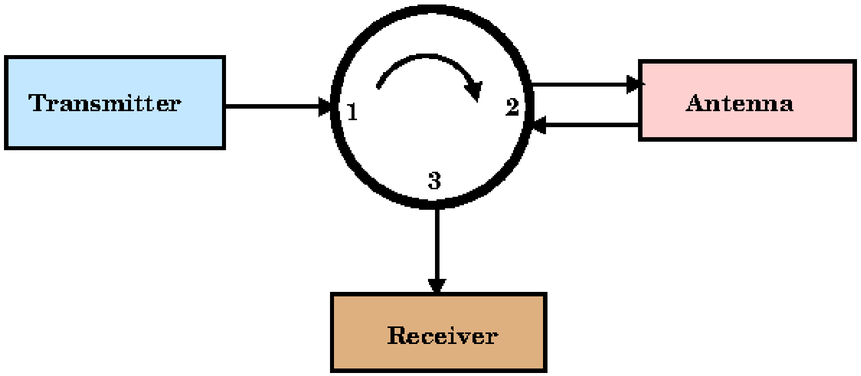


(b)

Nonreciprocal Three-Port Networks

- circulator often found in duplexer





Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}^{*}S_{23}=0$$

$$S_{12}^{*}S_{13}+S_{23}^{*}S_{33}=0$$

$$S_{23}^{*}S_{12}+S_{33}^{*}S_{13}=0$$

$$|S_{12}|^2+|S_{13}|^2=1$$

$$|S_{12}|^2+|S_{23}|^2=1$$

$$|S_{13}|^2+|S_{23}|^2+|S_{33}|^2=1$$

$$S_{13}=S_{23}=0$$

$$|S_{13}|=|S_{23}|$$

$$|S_{12}|=|S_{33}|=1$$

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network

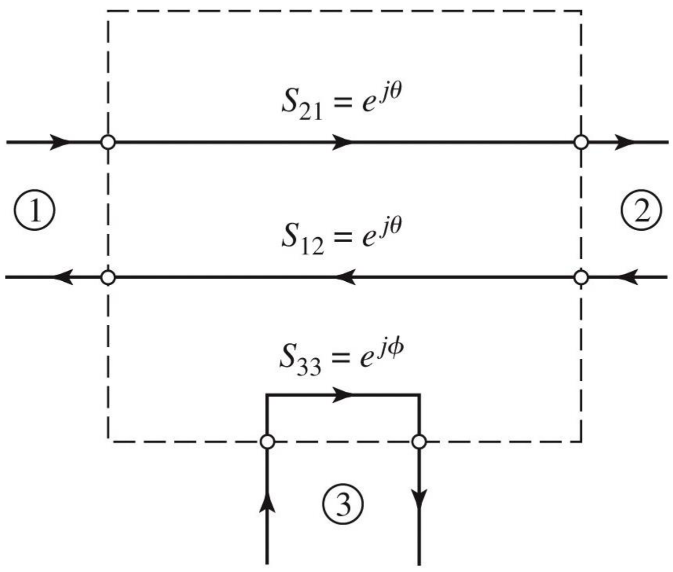
$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}=S_{23}=0 \qquad |S_{12}|=|S_{33}|=1$$

$$S_{12}=e^{j\theta}$$

$$S_{33}=e^{j\varphi}$$

$$[S]=\begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:
  - a matched two-port **line**
  - a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4x4 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Four-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0, S_{44}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} = 0 \quad / \cdot S_{24}^*$$
$$S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} = 0 \quad / \cdot S_{13}^*$$
$$S_{14}^* \cdot \left(\left|S_{13}\right|^2 - \left|S_{24}\right|^2 \right) = 0$$

$$S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} = 0 \quad / \cdot S_{12}$$
$$S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} = 0 \quad / \cdot S_{34}^*$$
$$S_{23} \cdot \left(\left|S_{12}\right|^2 - \left|S_{34}\right|^2 \right) = 0$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is **directional**

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$\left|S_{12}\right|^2 + \left|S_{13}\right|^2 = 1$$
$$\left|S_{12}\right|^2 + \left|S_{24}\right|^2 = 1$$
$$\Rightarrow \left|S_{13}\right| = \left|S_{24}\right|$$

$$\left|S_{13}\right|^2 + \left|S_{34}\right|^2 = 1$$
$$\left|S_{24}\right|^2 + \left|S_{34}\right|^2 = 1$$
$$\Rightarrow \left|S_{12}\right| = \left|S_{34}\right|$$

Four-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}| = |S_{34}| = \alpha \qquad |S_{13}| = |S_{24}| = \beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12} = S_{34} = \alpha \qquad S_{13} = \beta \cdot e^{j\theta} \qquad S_{24} = \beta \cdot e^{j\phi}$$

$$S_{12}^* \cdot S_{13} + S_{24}^* \cdot S_{34} = 0 \quad \rightarrow \quad \theta + \phi = \pi \pm 2 \cdot n \cdot \pi$$

$$|S_{12}|^2 + |S_{24}|^2 = 1 \quad \rightarrow \quad \alpha^2 + \beta^2 = 1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^* \cdot \left(|S_{13}|^2 - |S_{24}|^2\right) = 0 \qquad S_{23} \cdot \left(|S_{12}|^2 - |S_{34}|^2\right) = 0$$

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always** **directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler (90°) $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler (180°) $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler

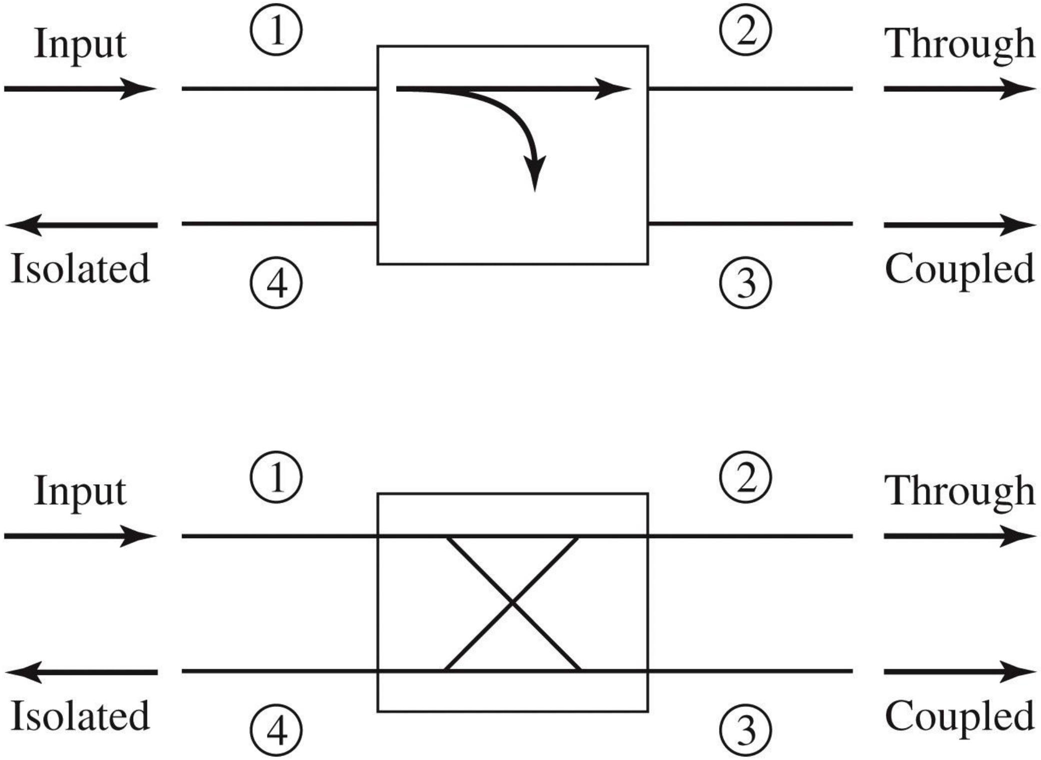


Figure 7.4

$$\left|S_{12}\right|^2 = \alpha^2 = 1 - \beta^2$$

$$\left|S_{13}\right|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

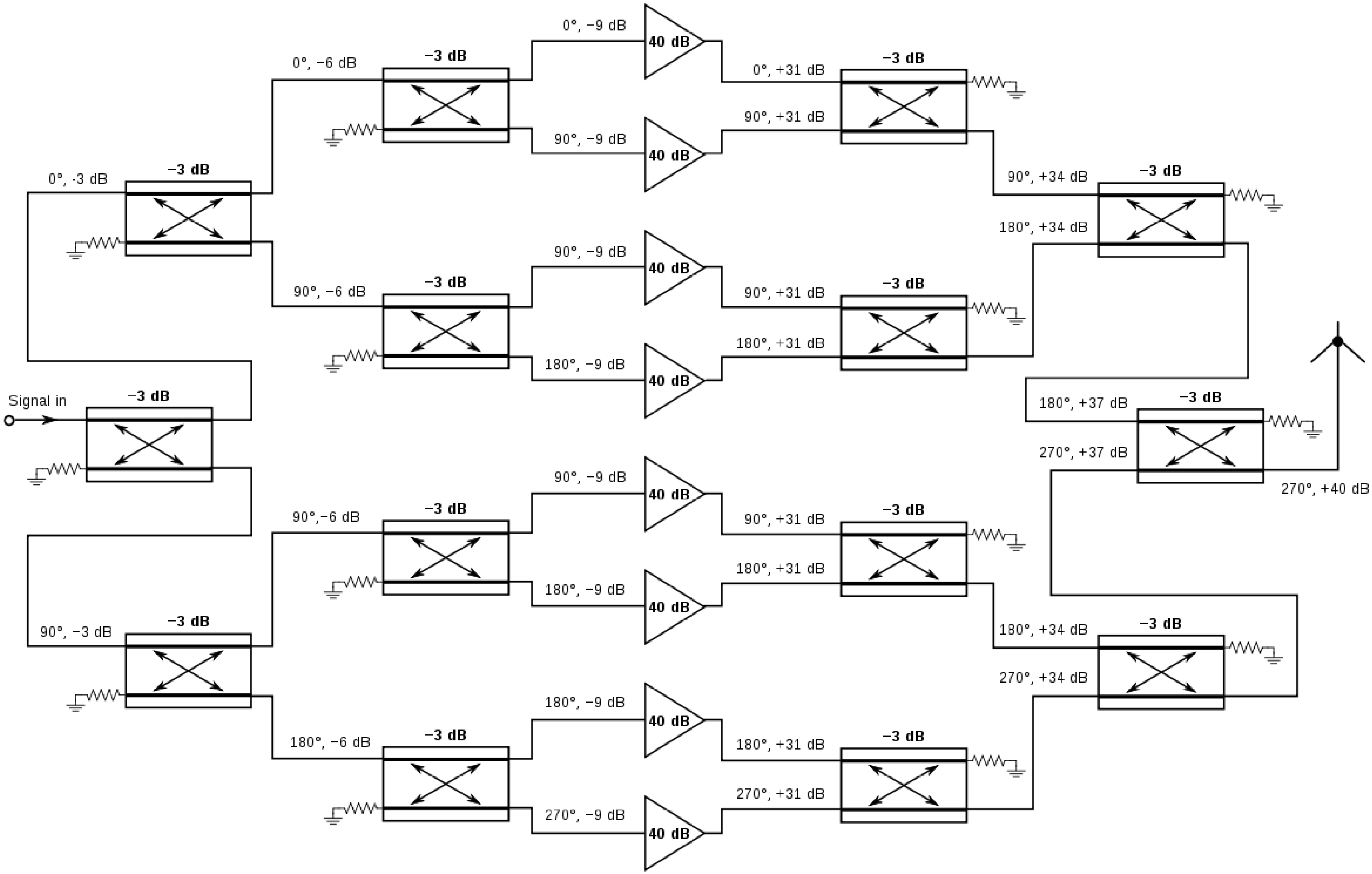
$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{\left|S_{14}\right|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log\left|S_{14}\right|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \left[dB\right]$$

Balanced amplifiers



Power dividers

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - **lossless**
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Power division of the T-junction

- consists in splitting an input line into two separate output lines
- available in various technologies for the lines

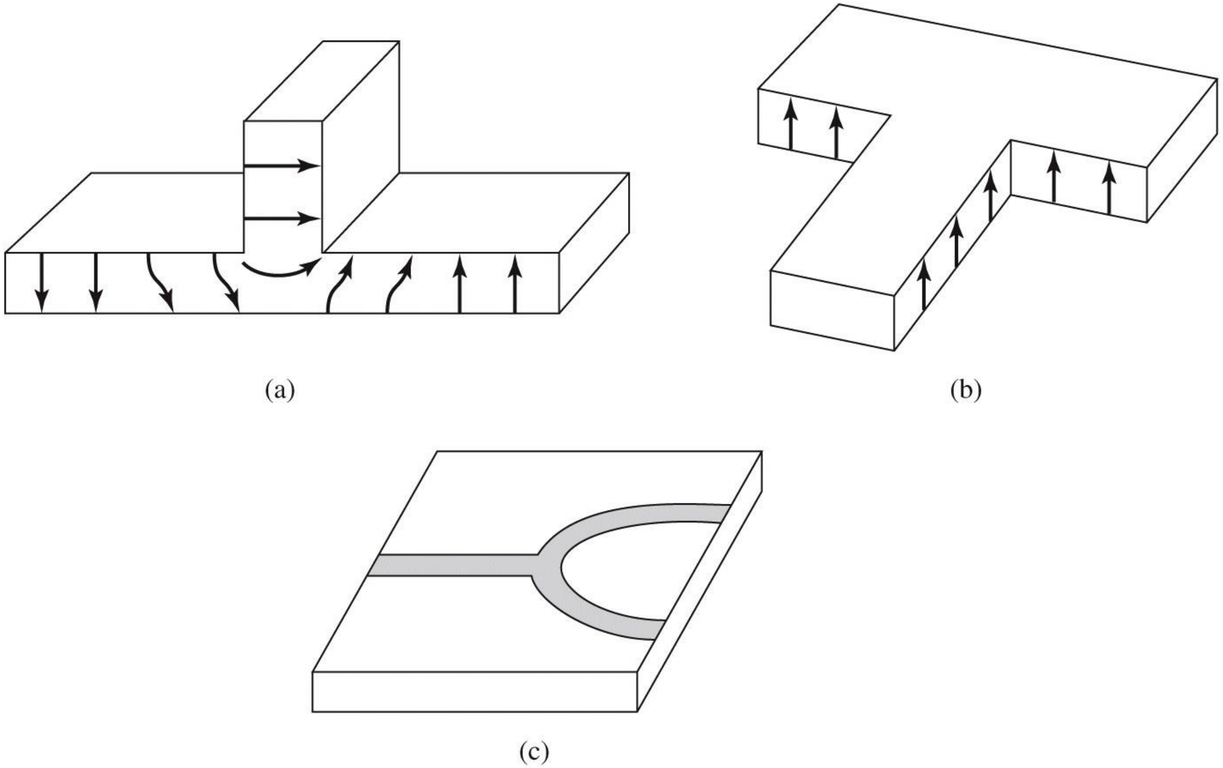


Figure 7.5

Power division of the T-junction

- if the lines are lossless, the network is reciprocal, so it cannot be matched at all ports simultaneously

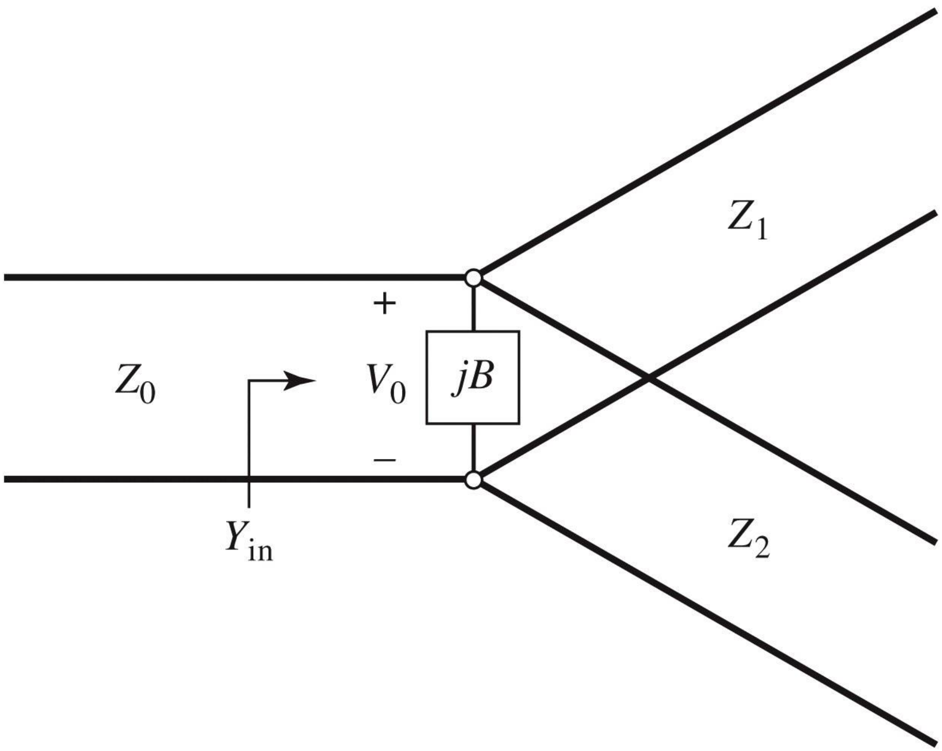


Figure 7.6

- there may be fringing fields and higher order modes associated with the discontinuity at such a junction
- the stored energy can be accounted for by a lumped susceptance: **B**
- Designing the power divider targets matching to the input line Z_o
  - outputs (unmatched, Z_1 and Z_2) can be, if needed, matched to Z_o ($\lambda/4$, binomial, Chebyshev)

Power division of the T-junction

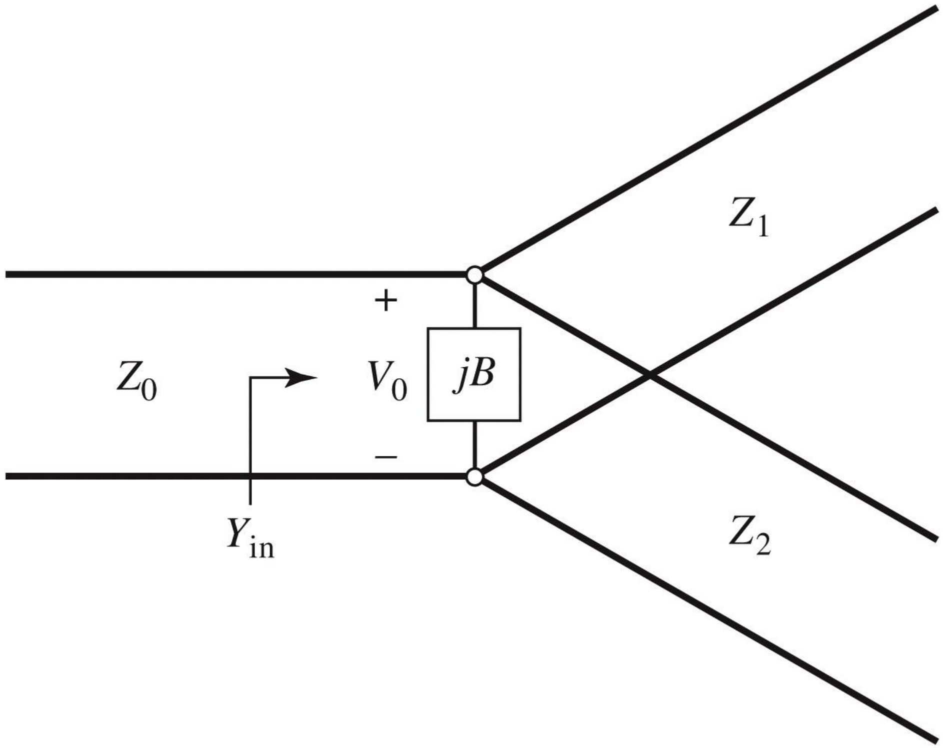


Figure 7.6

$$Y_{in} = j \cdot B + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

- If the transmission lines are assumed to be lossless, then the characteristic impedances are real
- the matching condition can be met only if **B ≅ 0** thus the matching condition is:

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

In practice, if **B** is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.

Power division of the T-junction

- if V_o is the voltage at the junction, we can compute how the input power is divided between the two output lines

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1}$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2}$$

then:

$$P_{in} = P_1 + P_2$$ (lossless/input matching)

$$\frac{P_1}{P_2} = \frac{Z_2}{Z_1} = \alpha$$ (power division between the two output lines)

$$P_1 = P_{in} \cdot \frac{Z_2}{Z_1 + Z_2} \qquad P_2 = P_{in} \cdot \frac{Z_1}{Z_1 + Z_2}$$

$$P_1 = P_{in} \cdot \frac{\alpha}{1+\alpha} \qquad P_2 = P_{in} \cdot \frac{1}{1+\alpha}$$

$$Z_1 = Z_0 \cdot \left(1 + \frac{1}{\alpha}\right) \qquad Z_2 = Z_0 \cdot (1+\alpha)$$

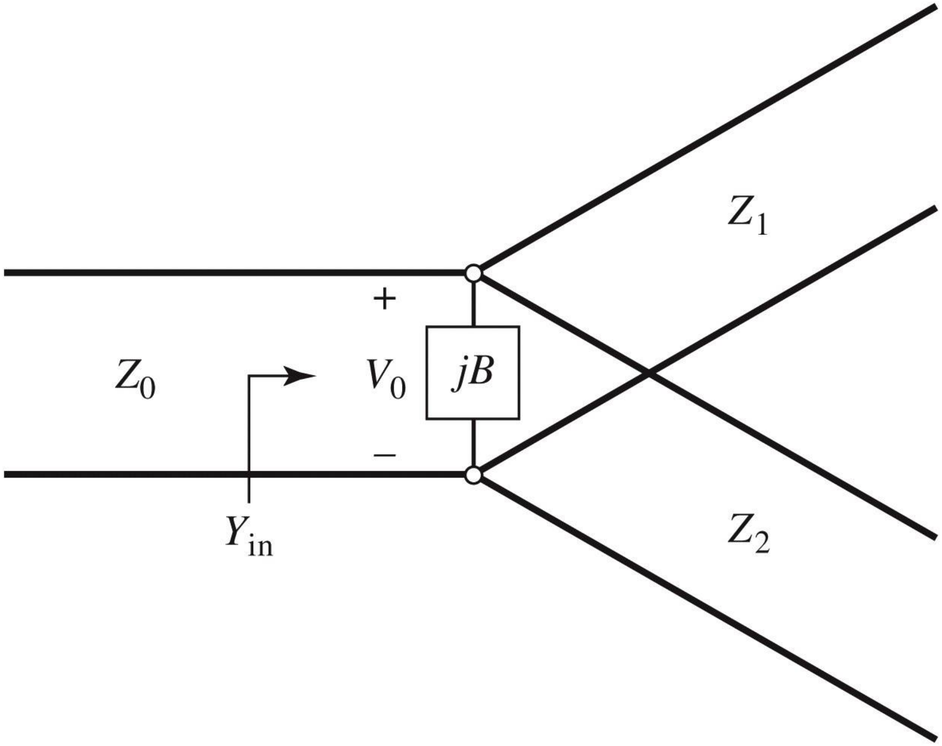


Figure 7.6

Power division of the T-junction

- S matrix
  - lossless (unitary matrix)
  - reciprocal (symmetrical matrix)
  - input port is matched $S_{11} = 0$

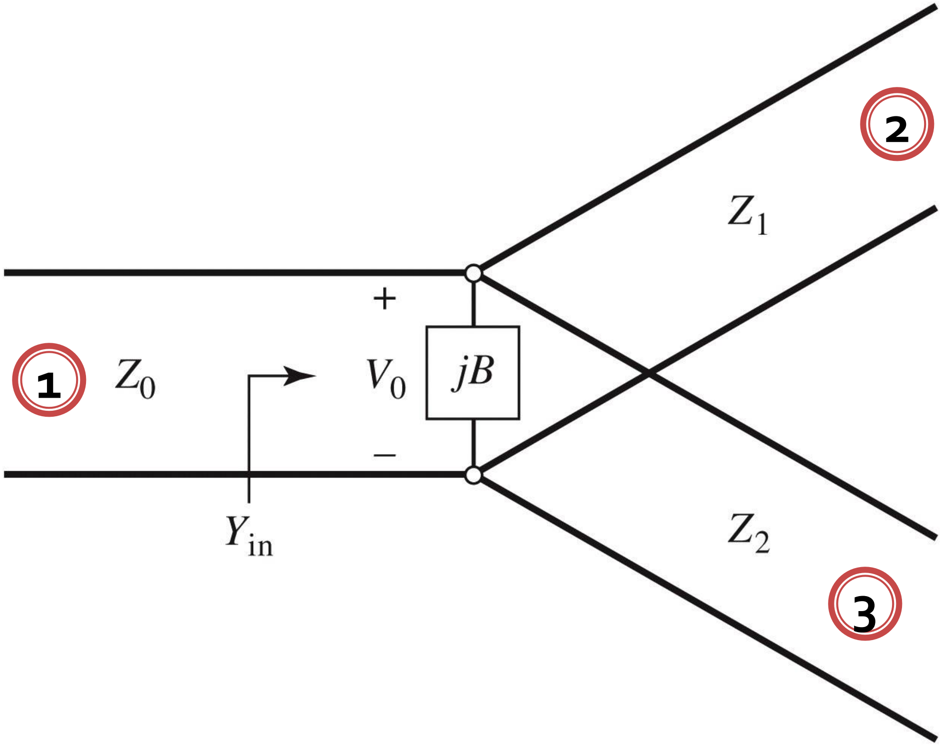


Figure 7.6

$$P_2 = P_1 \cdot \frac{\alpha}{1+\alpha} \qquad S_{21} = S_{12} = \sqrt{\frac{\alpha}{1+\alpha}}$$

$$P_3 = P_1 \cdot \frac{1}{1+\alpha} \qquad S_{31} = S_{13} = \sqrt{\frac{1}{1+\alpha}}$$

the reflection coefficients seen looking into the output ports

$$S_{22} = \Gamma_1 = \frac{Z_0 \parallel Z_2 - Z_1}{Z_0 \parallel Z_2 + Z_1} = -\frac{1}{1+\alpha}$$

$$S_{33} = \Gamma_2 = \frac{Z_0 \parallel Z_1 - Z_2}{Z_0 \parallel Z_1 + Z_2} = -\frac{\alpha}{1+\alpha}$$

Power division of the T-junction

$$[S]=\begin{bmatrix} 0 & \sqrt{\frac{\alpha}{1+\alpha}} & \sqrt{\frac{1}{1+\alpha}} \\ \sqrt{\frac{\alpha}{1+\alpha}} & -\frac{1}{1+\alpha} & x \\ \sqrt{\frac{1}{1+\alpha}} & x & -\frac{\alpha}{1+\alpha} \end{bmatrix}$$

Unitary matrix, columns 1 and 2

$$0-\frac{1}{1+\alpha}\cdot\sqrt{\frac{\alpha}{1+\alpha}}+x\cdot\sqrt{\frac{1}{1+\alpha}}=0$$

$$S_{23}=S_{32}=\frac{\sqrt{\alpha}}{1+\alpha}$$

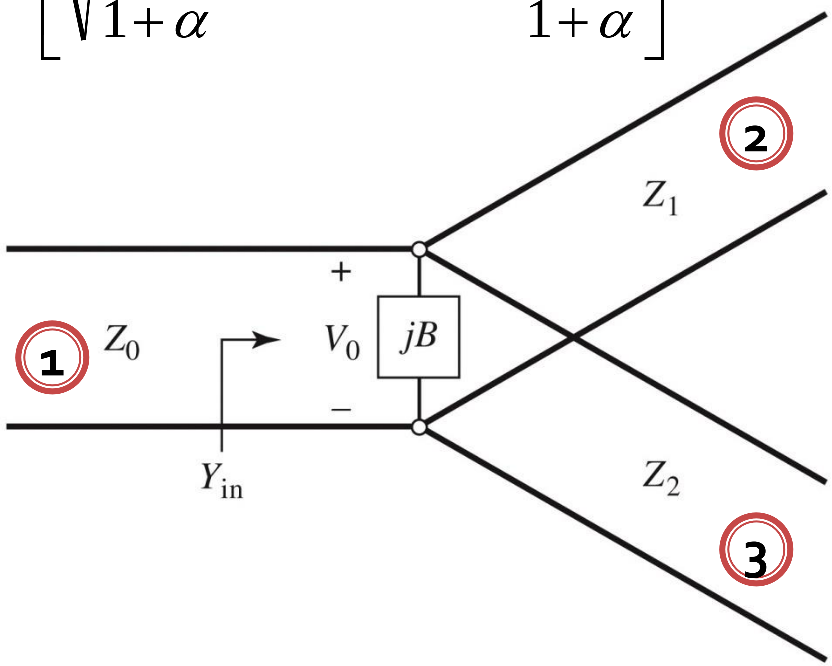


Figure 7.6

$$[S]=\begin{bmatrix} 0 & \sqrt{\frac{\alpha}{1+\alpha}} & \sqrt{\frac{1}{1+\alpha}} \\ \sqrt{\frac{\alpha}{1+\alpha}} & -\frac{1}{1+\alpha} & \frac{\sqrt{\alpha}}{1+\alpha} \\ \sqrt{\frac{1}{1+\alpha}} & \frac{\sqrt{\alpha}}{1+\alpha} & -\frac{\alpha}{1+\alpha} \end{bmatrix}$$

Power division of the T-junction

- 3dB divider
  - equal splitting of the power between the two outputs
  - $Z_1 = Z_2 = 2 \cdot Z_0$, $\alpha = 1$

$$[S]=\begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

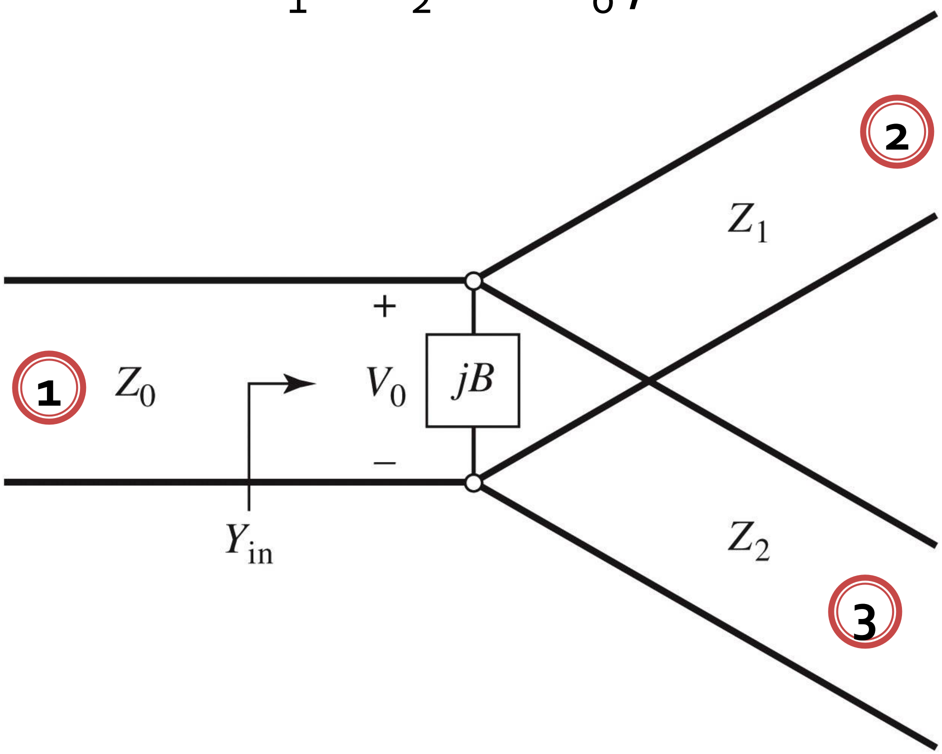


Figure 7.6

If we add $\lambda/4$ transformers to match outputs to Z_0 S matrix:

$$[S]=\begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{j}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Example

- Design a lossless T-junction divider with a 30Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30Ω. (**Pozar** problem)

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0}$$

$$\begin{cases} P_1 + P_2 = P_{in} \\ P_1 : P_2 = 3:1 \end{cases} \Rightarrow \begin{cases} P_1 = \frac{1}{4} \cdot P_{in} \\ P_2 = \frac{3}{4} \cdot P_{in} \end{cases}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1} = \frac{1}{4} \cdot P_{in} \qquad Z_1 = 4 \cdot Z_0 = 120\,\Omega$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2} = \frac{3}{4} \cdot P_{in} \qquad Z_2 = 4 \cdot Z_0 / 3 = 40\,\Omega$$

Input match check

$$Z_{in} = 40\,\Omega \,||\, 120\,\Omega = 30\,\Omega$$

quarter-wave transformers $Z_c^i = \sqrt{Z_i \cdot Z_L}$

$$Z_c^1 = \sqrt{Z_1 \cdot Z_L} = \sqrt{120\Omega \cdot 30\Omega} = 60\Omega \qquad Z_c^2 = \sqrt{Z_2 \cdot Z_L} = \sqrt{40\Omega \cdot 30\Omega} = 34.64\Omega$$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports

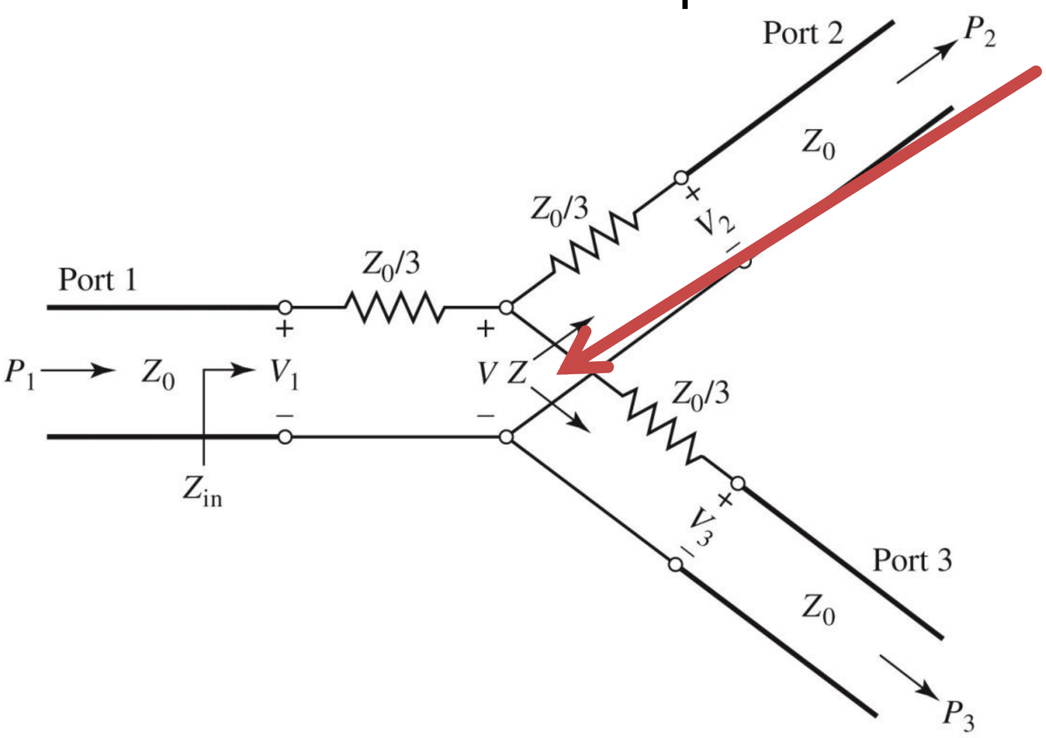


Figure 7.7

The impedance Z, seen looking into the Zo/3 resistor followed by a terminated output line:

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

The input line will be terminated with a Zo/3 resistor in series with two such lines Z in parallel

$$Z_{in} = \frac{Z_0}{3} + \frac{1}{2} \cdot \frac{4Z_0}{3} = Z_0$$

so it will be matched: $S_{11} = 0$

from symmetry: $S_{11} = S_{22} = S_{33} = 0$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

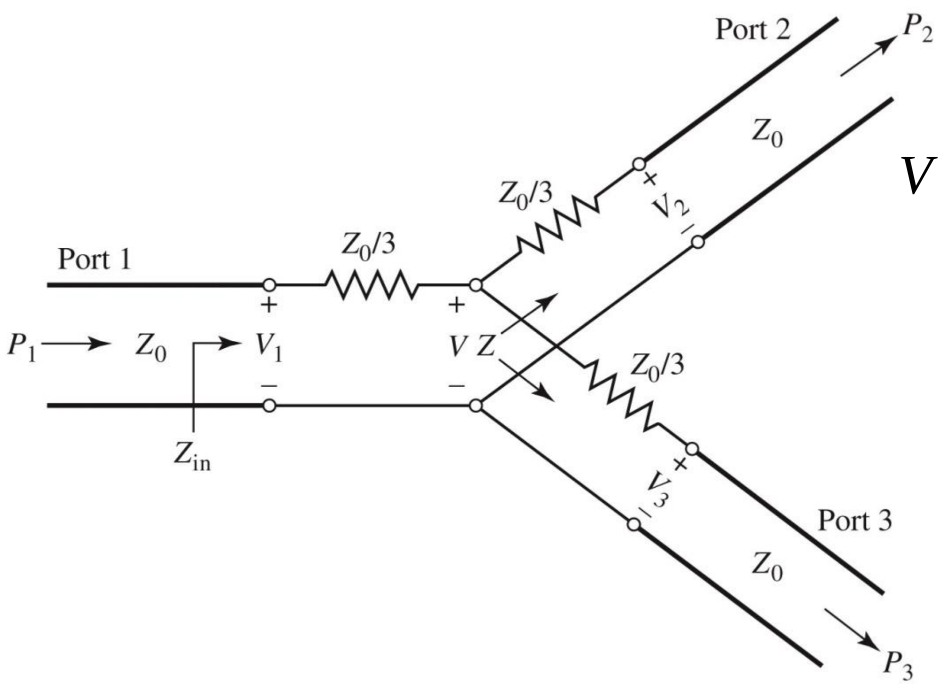


Figure 7.7

If the voltage at port 1 is V1, then by voltage division the voltage V at the junction is:

$$V = V_1 \cdot \frac{Z/2}{Z/2 + Z_0/3} = V_1 \cdot \frac{2Z_0/3}{2Z_0/3 + Z_0/3} = \frac{2}{3} \cdot V_1$$

The output voltages are, again by voltage division :

$$V_2 = V_3 = V \cdot \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4} \cdot V = \frac{1}{2} \cdot V_1$$

$$S_{21} = S_{31} = \frac{1}{2}$$

from symmetry: $S_{21} = S_{31} = S_{23} = \frac{1}{2}$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal (S matrix is symmetrical) $S_{21} = S_{31} = S_{23} = \frac{1}{2}$
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

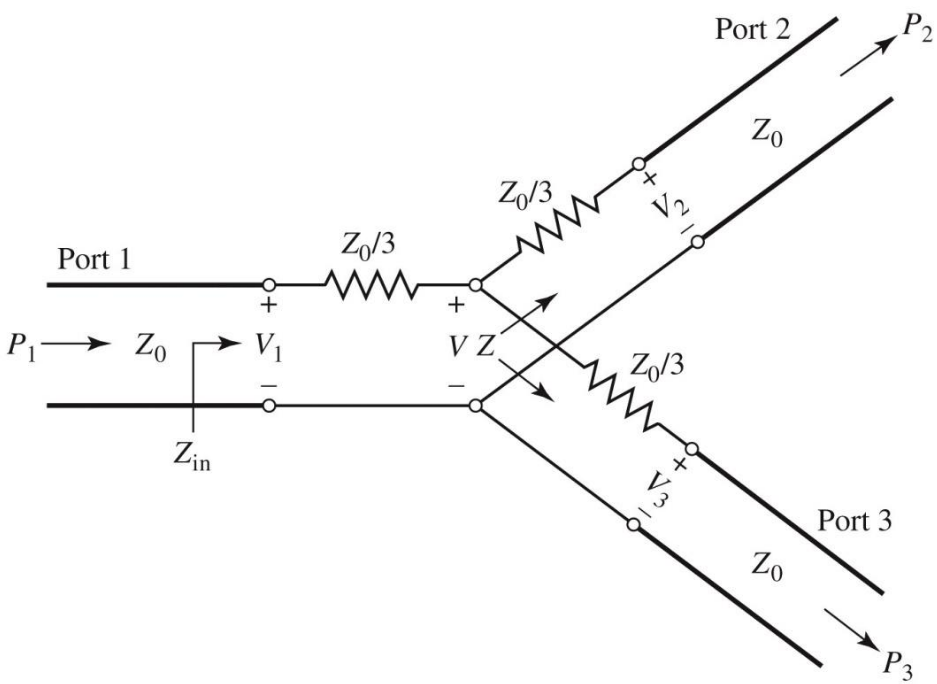


Figure 7.7

S matrix: $$[S] = \frac{1}{2} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

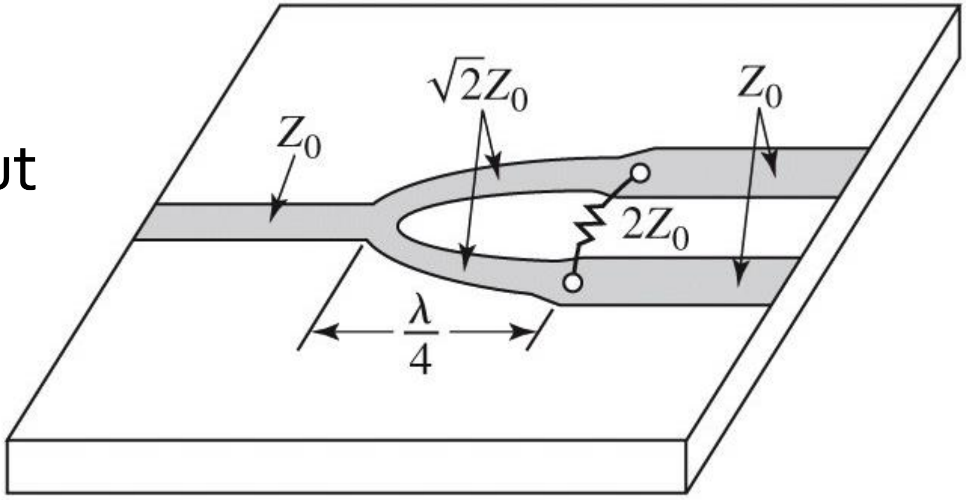
Powers: $$P_{in} = \frac{1}{2} \cdot \frac{V_1^2}{Z_0}$$

$$P_2 = P_3 = \frac{1}{2} \cdot \frac{(1/2 V_1)^2}{Z_0} = \frac{1}{8} \cdot \frac{V_1^2}{Z_0} = \frac{1}{4} \cdot P_{in}$$

Half of the supplied power is dissipated in the 3 resistors. The output powers are 6 dB below the input power level

The Wilkinson power divider

- Previous power dividers suffer from a major drawback, there is not isolation between the two output ports $S_{23} = S_{32} \neq 0$
  - this requirement is important in some applications
- The Wilkinson power divider solves this problem
  - it also has the useful property of appearing **lossless** when the output ports are matched
  - only reflected power from the output ports is dissipated



The Wilkinson power divider

- one input line
- two λ/4 transformers
- one resistor between the output lines

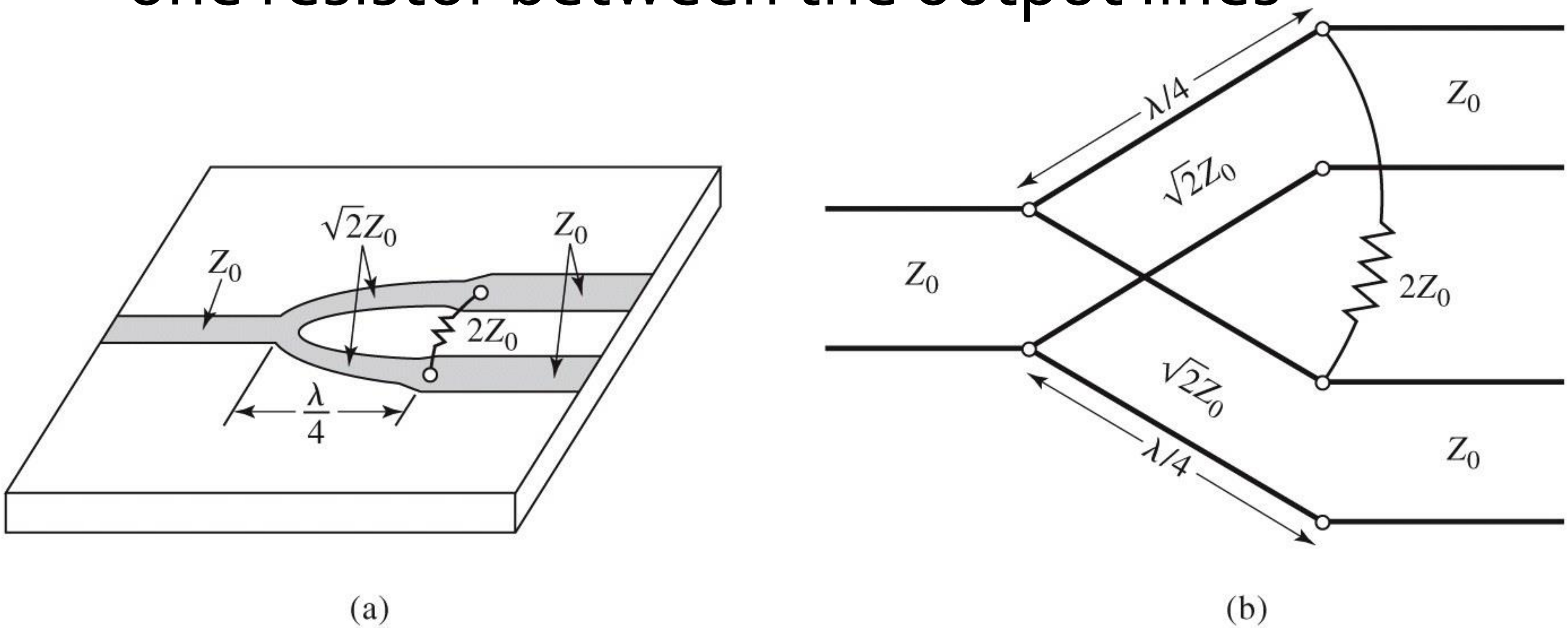


Figure 7.8

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

The Wilkinson power divider

- the circuit in normalized and symmetric form

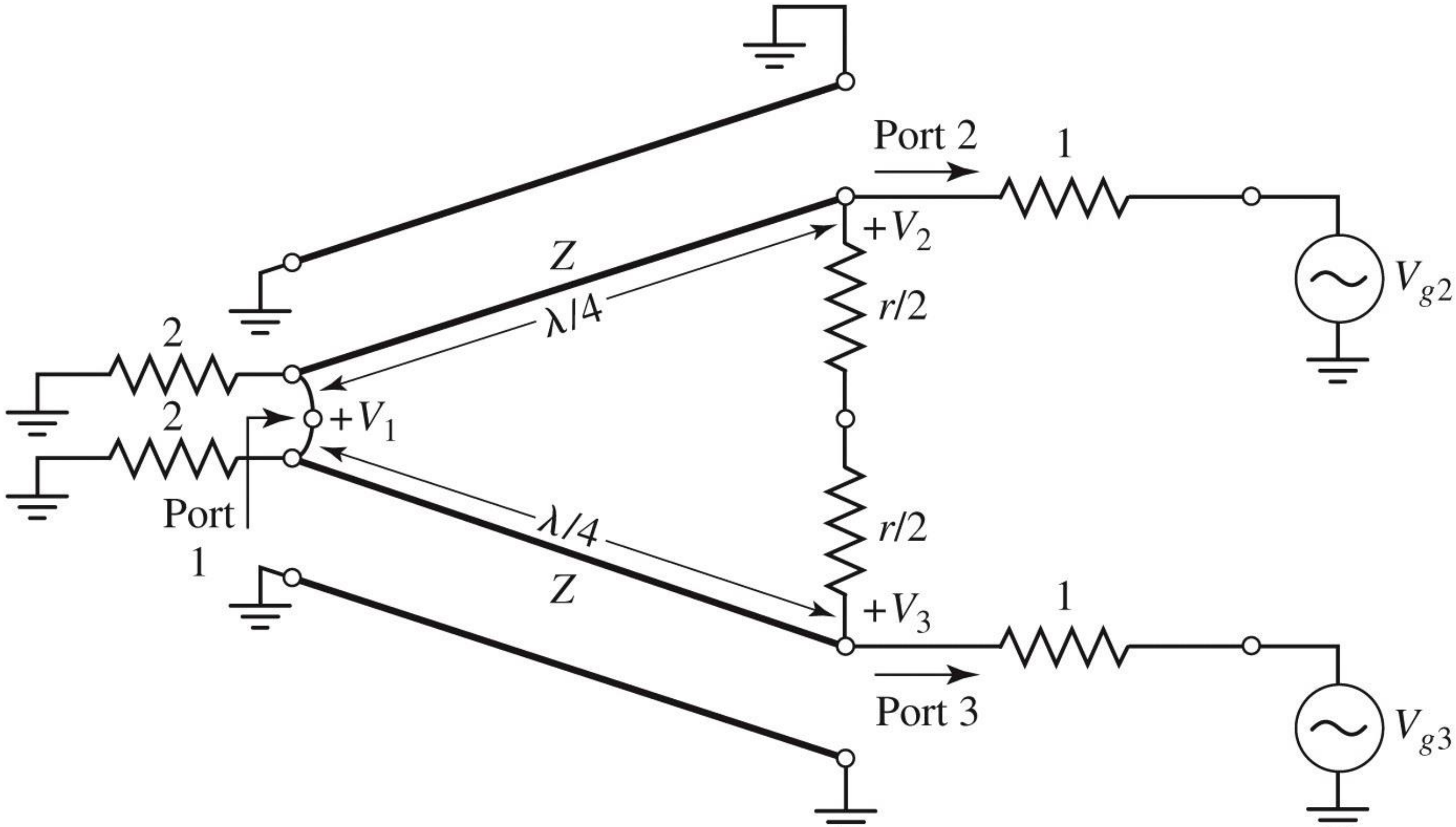
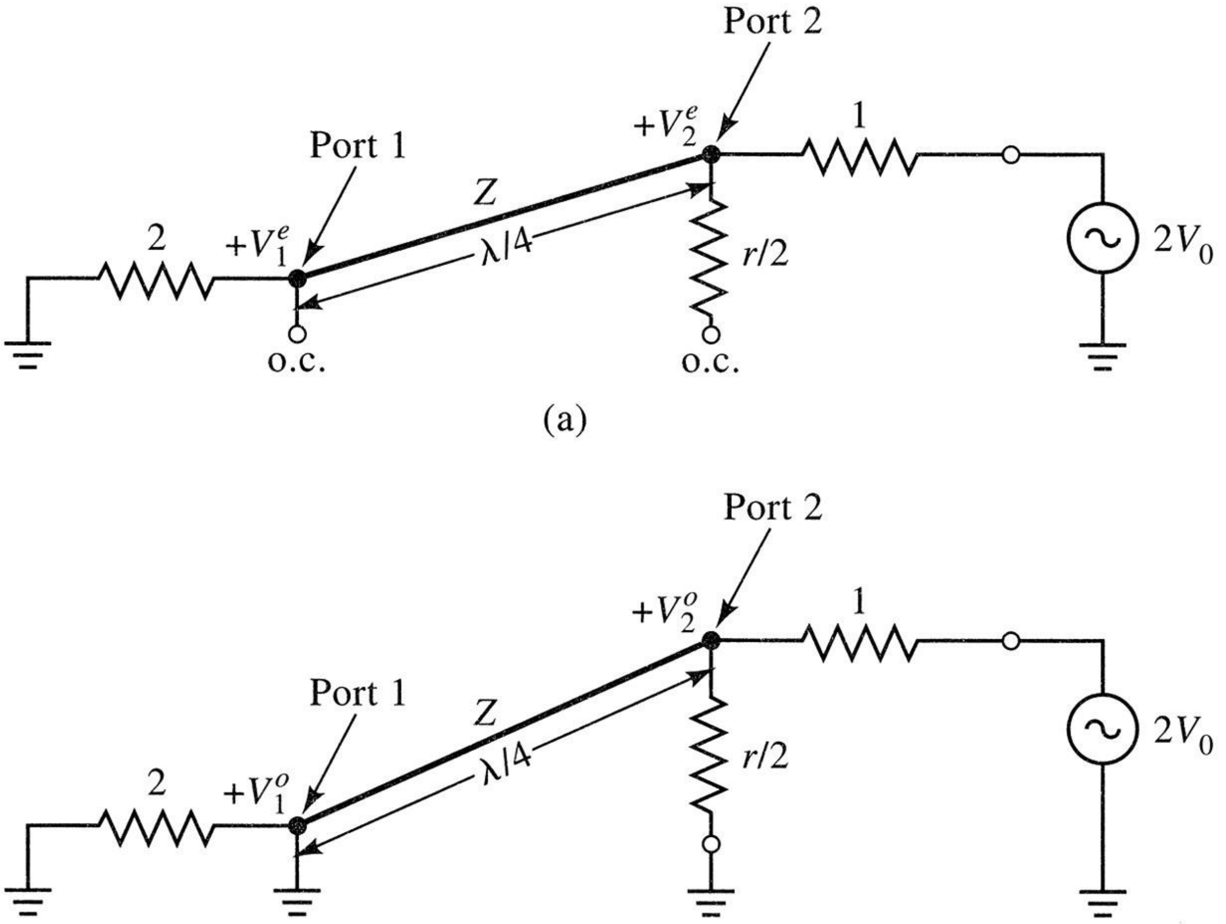


Figure 7.9

The Wilkinson power divider

- Even/Odd Mode Analysis

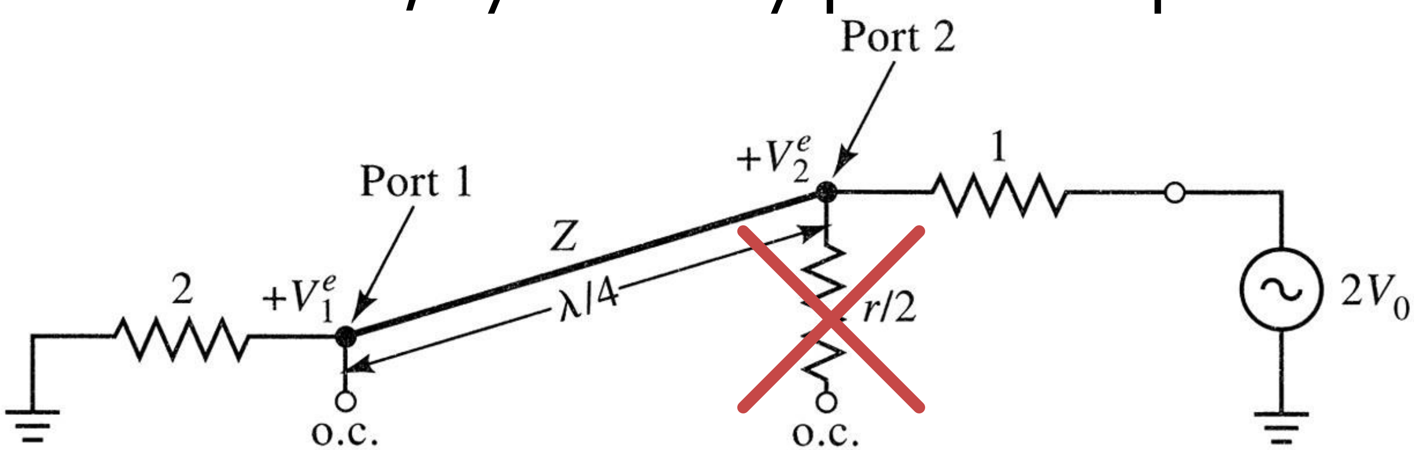


(a)

(b)

The Wilkinson power divider

- even mode, symmetry plane is open circuit



looking into port 2, λ/4 transformer with 2 load $Z_{in2}^e = \frac{Z^2}{2}$ if $Z = \sqrt{2}$ port 2 is matched $Z_{in2}^e = 1$

$$V(x) = V^+ \cdot \left(e^{-j\beta \cdot x} + \Gamma \cdot e^{j\beta \cdot x}\right)$$

x=0 at port 1

x=-λ/4 at port 2

$$V_2^e = V(-\lambda/4) = jV^+ \cdot (1-\Gamma) = V_0$$

$$Z_{in2}^e = 1$$

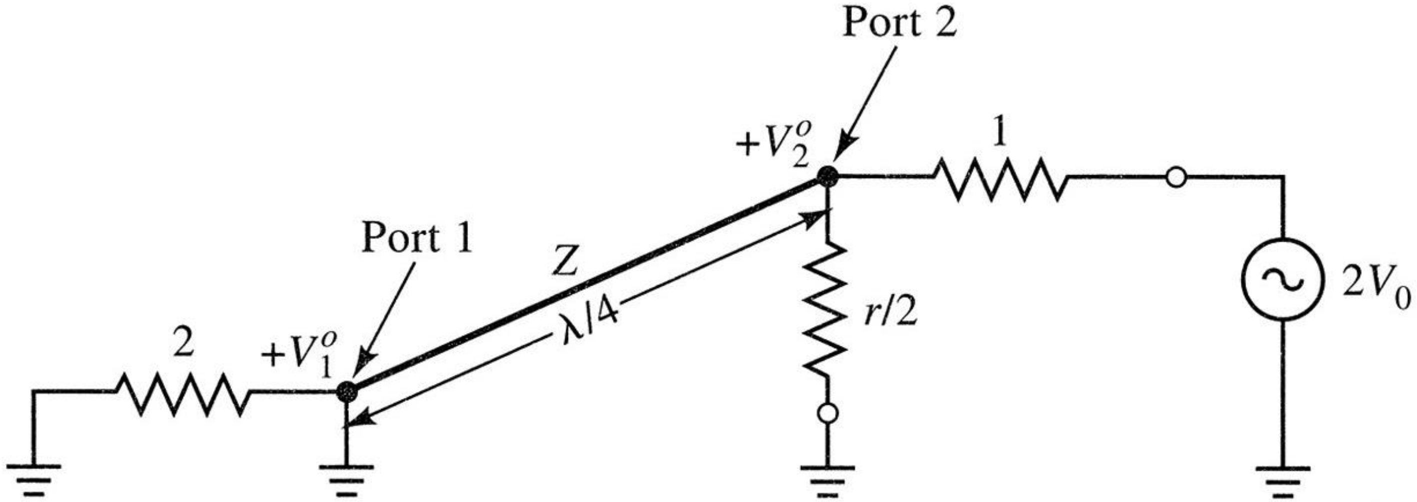
$$V_1^e = V(0) = V^+ \cdot (1+\Gamma) = jV_0 \cdot \frac{\Gamma+1}{\Gamma-1}$$

Γ: reflection coefficient seen at port 1 looking toward the resistor of normalized value 2 from the transformer $Z = \sqrt{2}$

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}} \qquad V_1^e = -jV_0\sqrt{2}$$

The Wilkinson power divider

- odd mode, symmetry plane is grounded



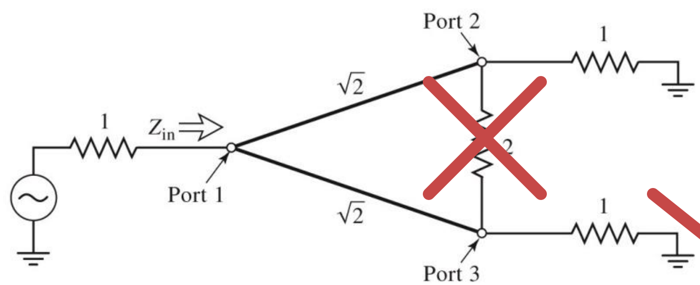
looking from port 2 the λ/4 line is short-circuited, impedance seen from port 2 is ∞ $Z_{in2}^o = r/2$ if $r = 2$ port 2 is matched

$Z_{in2}^o = 1 \rightarrow V_2^o = V_0$

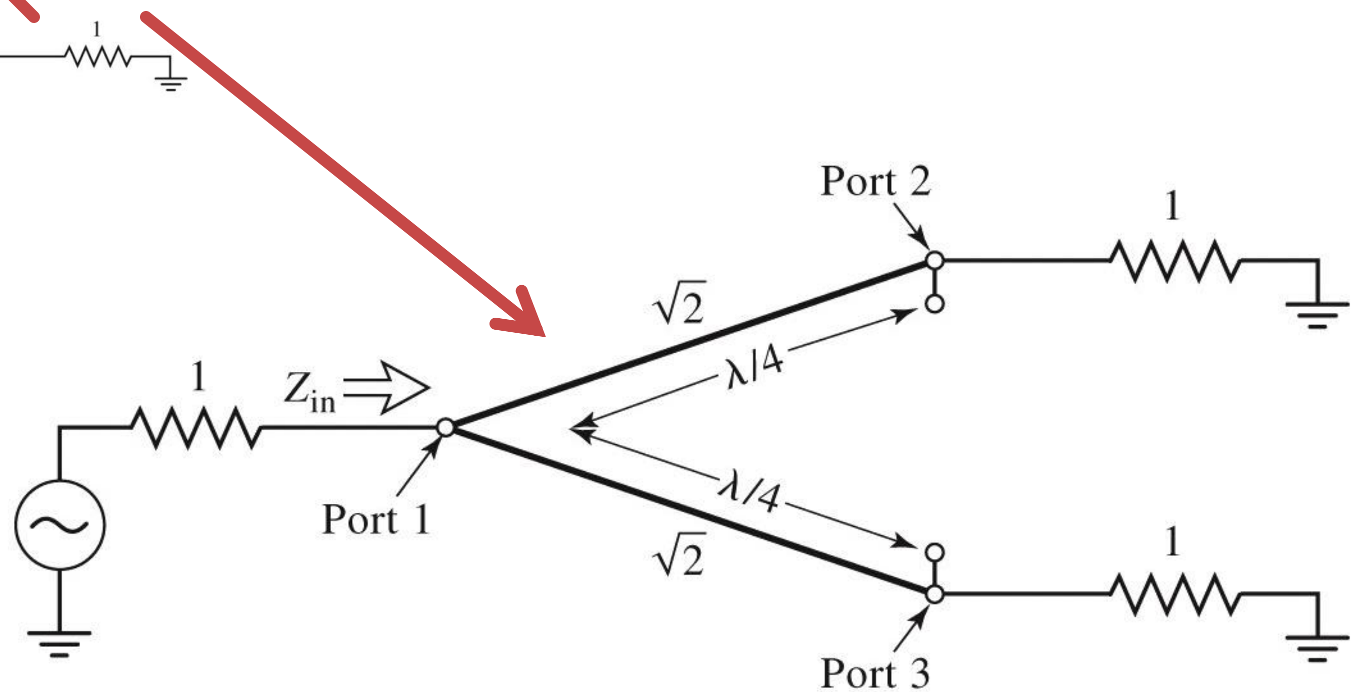
$V_1^o = 0$ in the odd mode all the power is dissipated in the *r*/2 resistor

The Wilkinson power divider

- input impedance in port 1



two λ/4 transformers with load 1 in parallel



$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$$

The Wilkinson power divider

- S parameters

$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1 \qquad S_{11} = 0$$

$$Z^{e}_{in2} = 1 \quad Z^{o}_{in2} = 1$$ and $$Z^{e}_{in3} = 1 \quad Z^{o}_{in3} = 1 \qquad S_{22} = S_{33} = 0$$

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -\frac{j}{\sqrt{2}}$$

and $S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$

$S_{23} = S_{32} = 0$ due to short or open at bisection, both eliminate transfer between the ports + reciprocal circuit

The Wilkinson power divider

- at design frequency (length of the transformer equal to $\lambda_o/4$) we have **isolation** between the two output ports

$$[S]=\begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & 0 & 0 \\ -\frac{j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

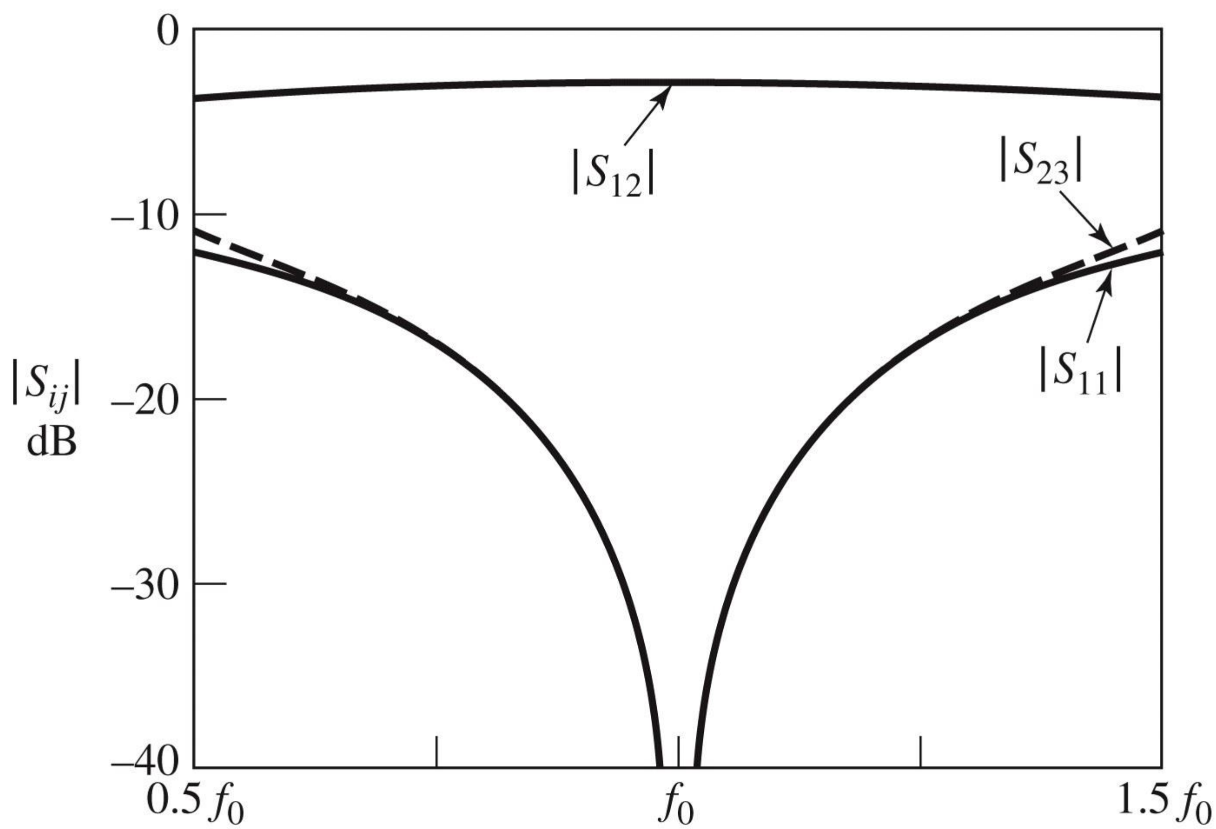


Figure 7.12

The Wilkinson power divider

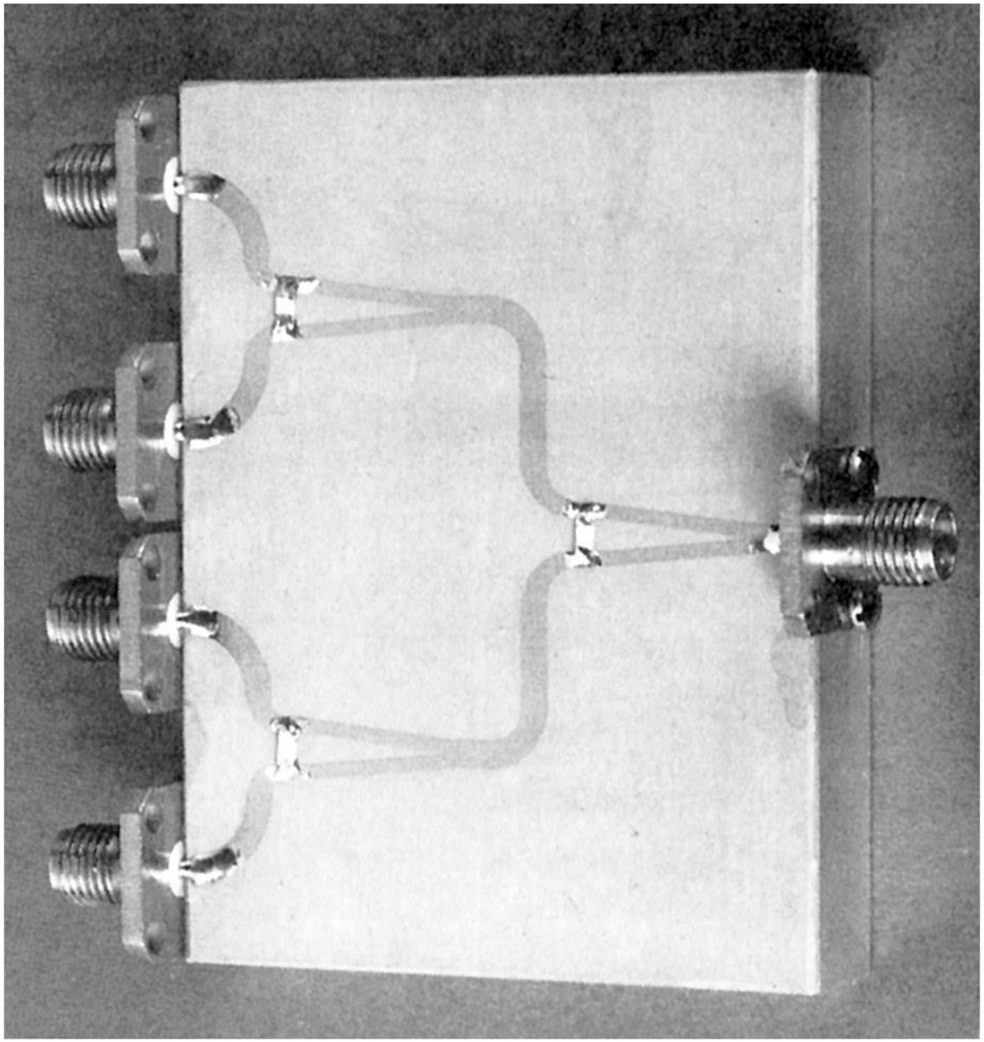
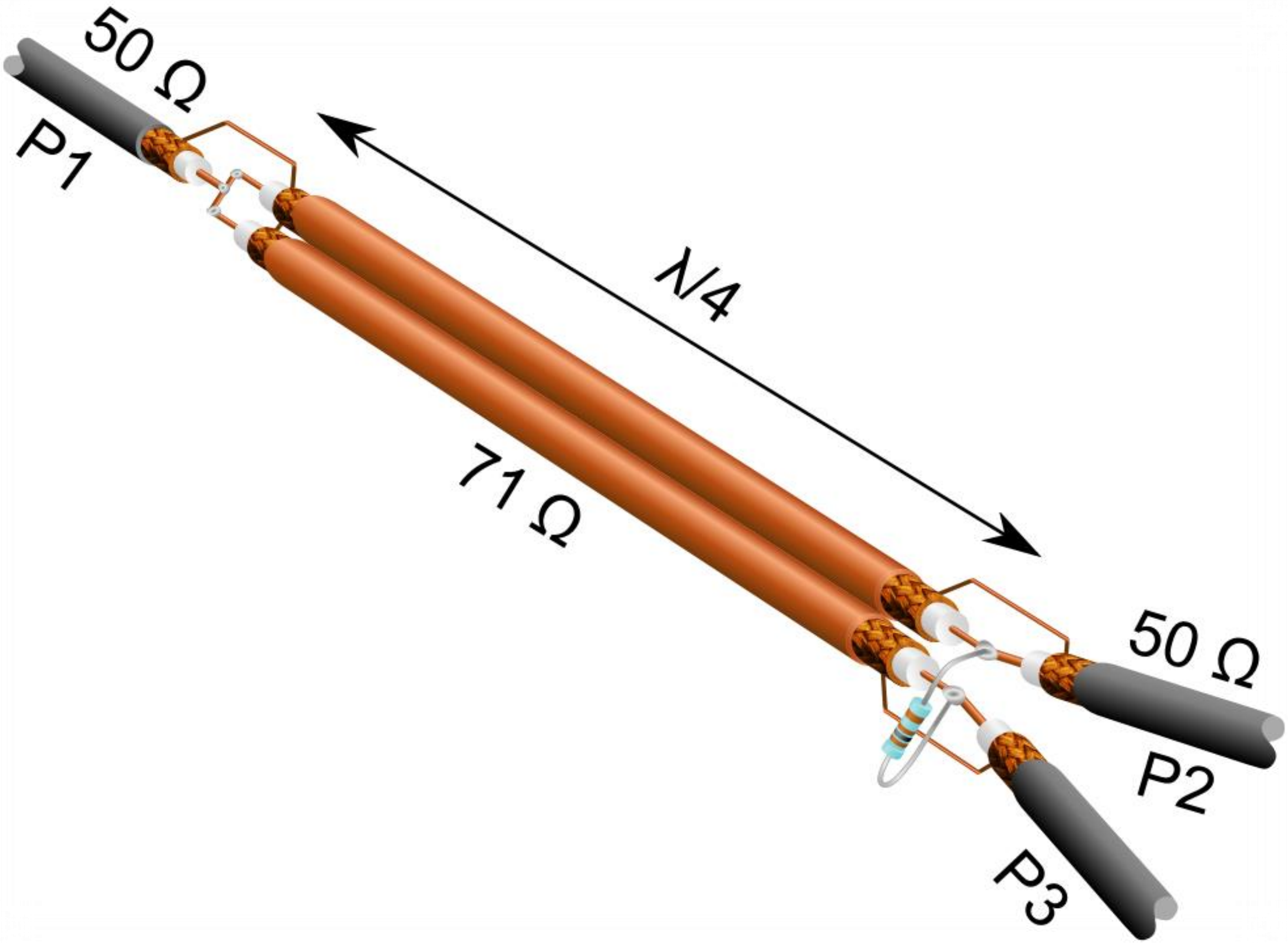


Figure 7.15
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

- 3 X Wilkinson = 4-way power divider

The Wilkinson power divider



Directional couplers

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always** **directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Directional Coupler

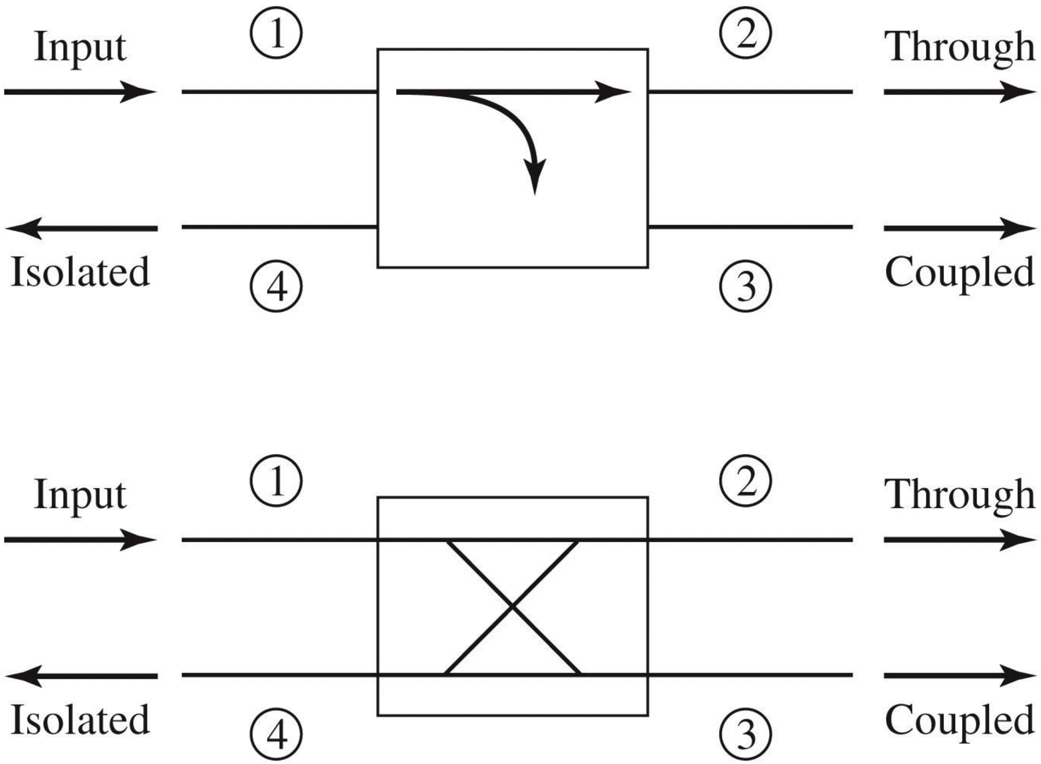


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ [dB]$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Hybrid Couplers

Hybrid Couplers are directional couplers with 3 dB coupling factor

$$\alpha = \beta = 1/\sqrt{2}$$

The cuadrature (90°) hybrid

$$\left(\theta = \phi = \pi/2\right)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

The 180° ring hybrid (rat-race)

$$\left(\theta = 0\,, \phi = \pi\right)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

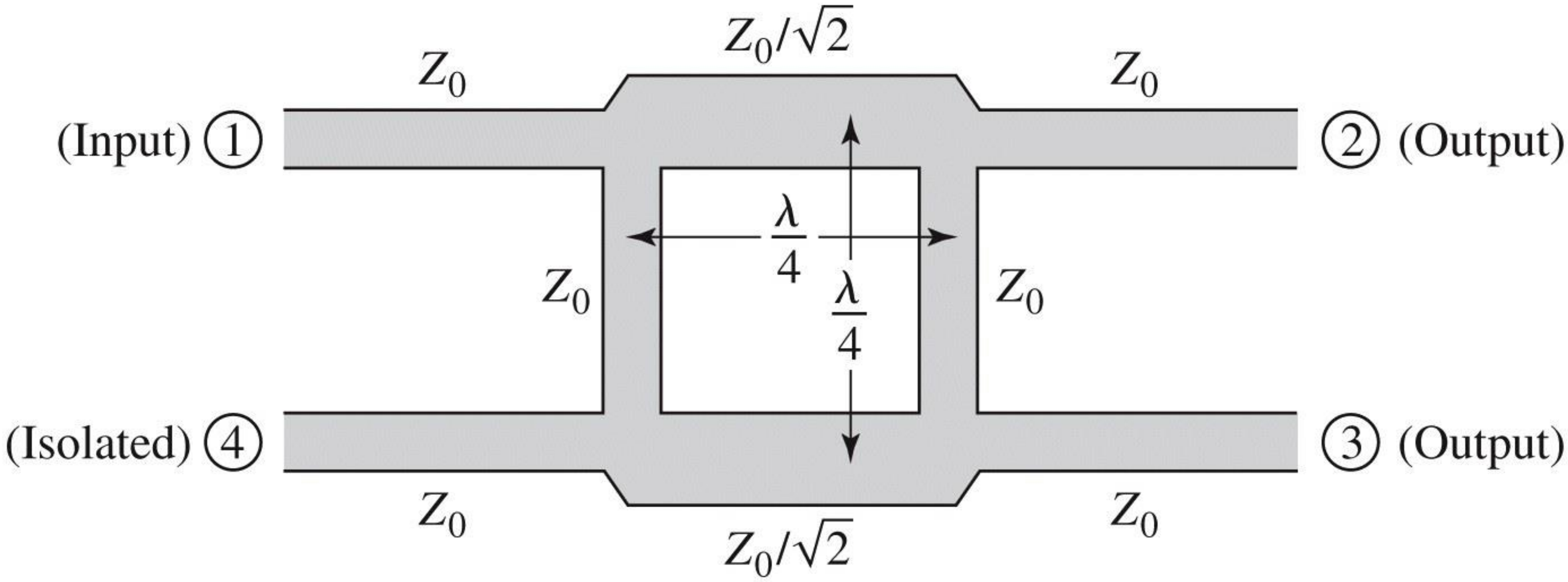
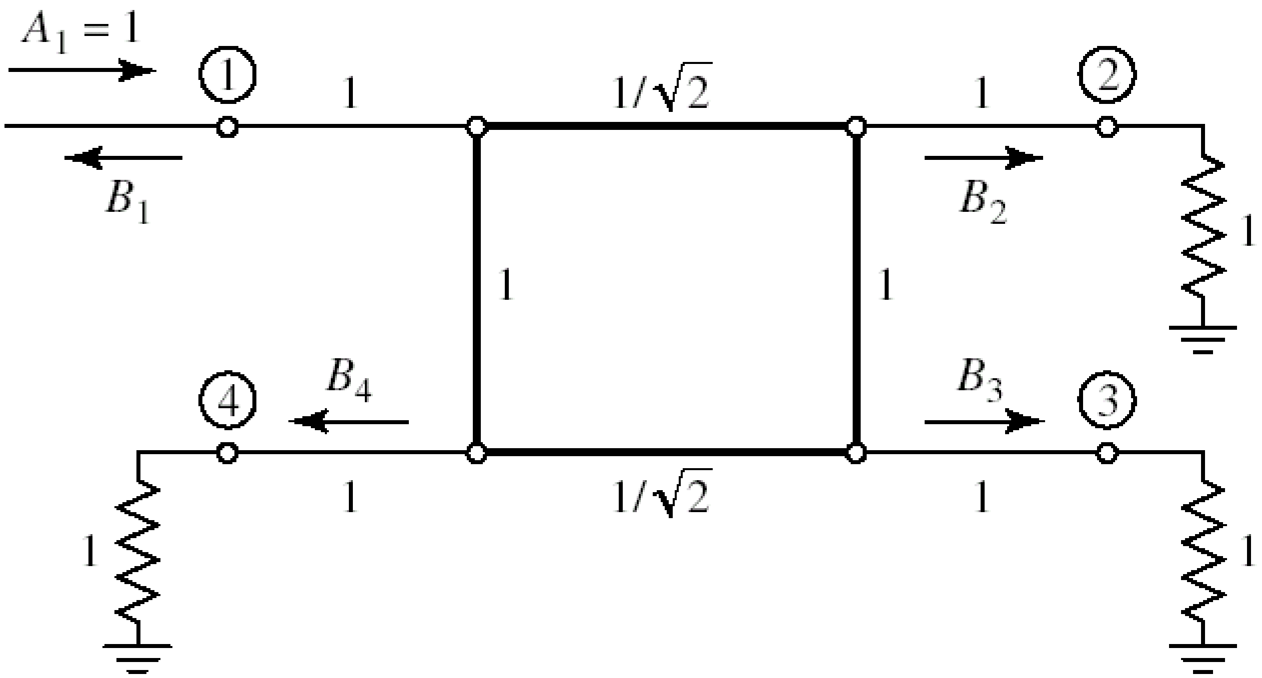


Figure 7.21

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Even/Odd Mode Analysis



Even/Odd Mode Analysis

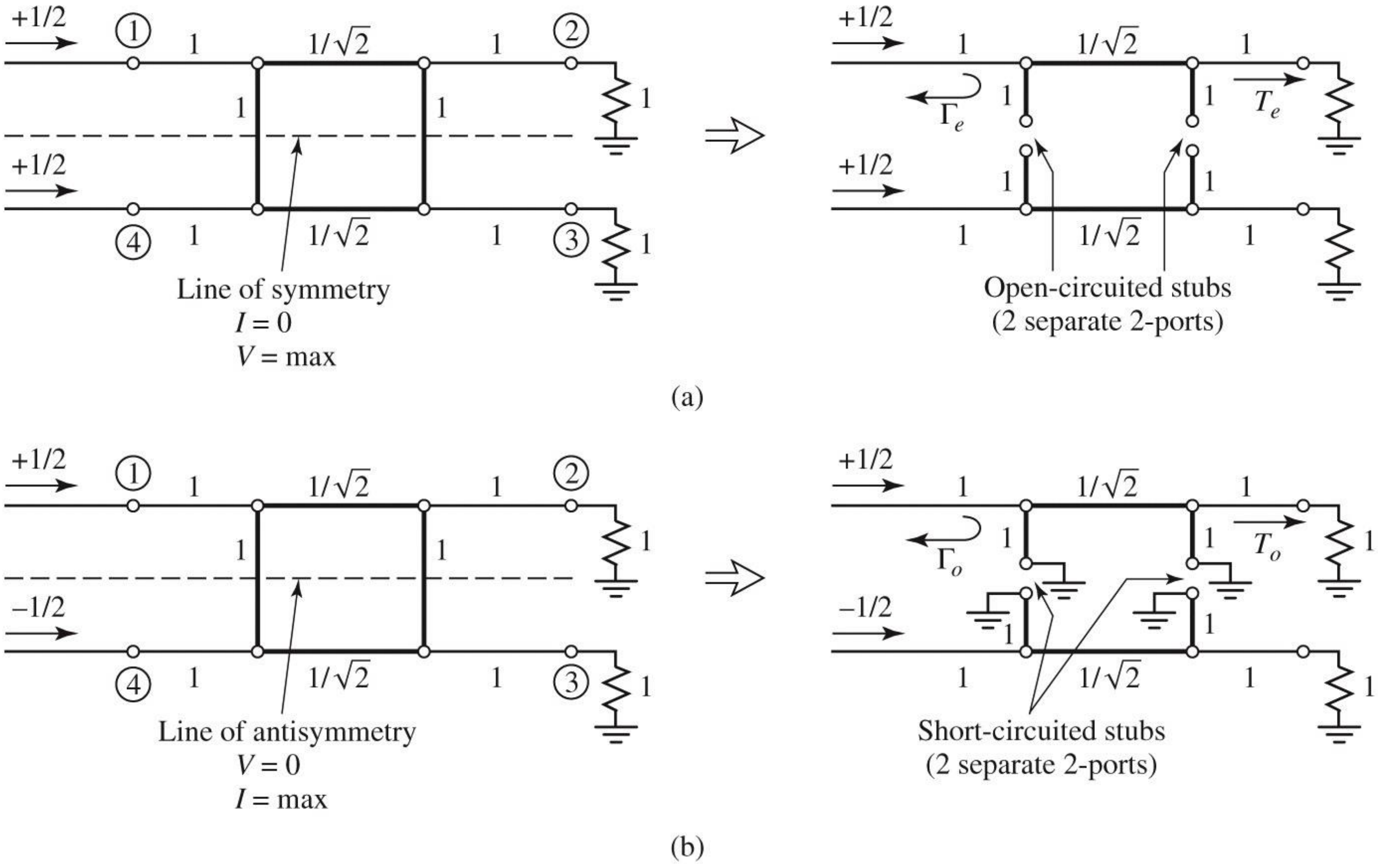
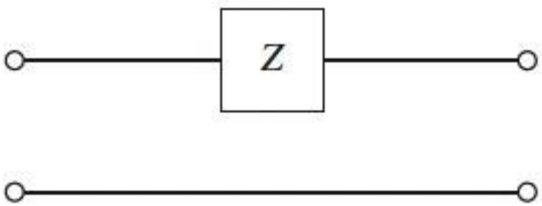
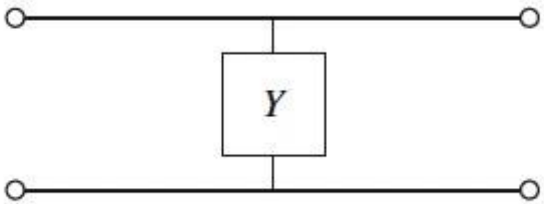
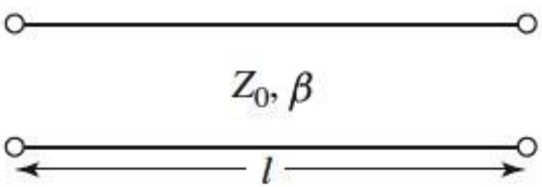


Figure 7.23

$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

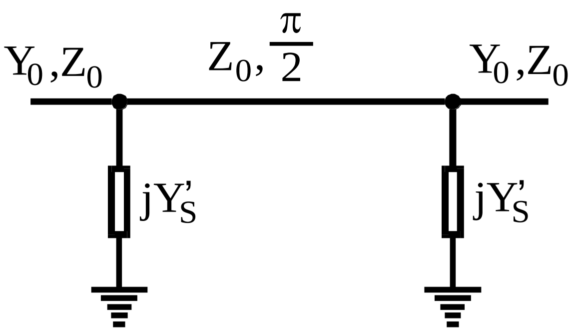
Library of ABCD matrices

TABLE 4.1 ***ABCD* Parameters of Some Useful Two-Port Circuits**

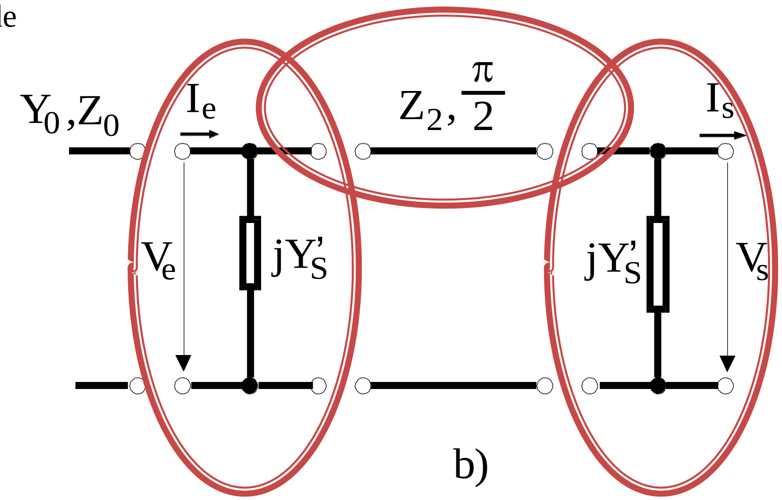
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta\ell$ $C = jY_0 \sin \beta\ell$	$B = jZ_0 \sin \beta\ell$ $D = \cos \beta\ell$

S parameters (from ABCD)

$$Y'_S = \begin{cases} Y_1 & \text{even mode} \\ -Y_1 & \text{odd mode} \end{cases}$$



a)



b)

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & jZ_2 \\ jY_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} -Y'_S Z_2 & jZ_2 \\ -jY'^2_S Z_2 + jY_2 & -Y'_S Z_2 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$S_{11} = \frac{j\frac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S + jY_2\right)}$$

$$S_{12} = \frac{2\left[\left(-Y'_S Z_2\right)^2 - jZ_2\left(-jY'^2_S Z_2 + jY_2\right)\right]}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$\Gamma = S_{11} = \frac{j\left(z_2 - y_2 + y'^2_S z_2\right)}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{22}$$

$$S_{21} = \frac{2}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$S_{22} = \frac{j\frac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\frac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$T = S_{21} = \frac{2}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{12}$$

Relation between two port S parameters and ABCD parameters

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}}\,\frac{\left(1 + S_{11} - S_{22} - \Delta S\right)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}}\,\frac{\left(1 + S_{11} + S_{22} + \Delta S\right)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}}\,\frac{1 - S_{11} - S_{22} + \Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}}\,\frac{1 - S_{11} + S_{22} - \Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02} + B - CZ_{01}Z_{02} - DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{12} = \frac{2(AD - BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02} + B - CZ_{01}Z_{02} + DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

Matching and coupling factor

$$\Gamma_e = \frac{j\cdot(z_2 - y_2 + y_1^2 z_2)}{-2y_1z_2 + j(z_2 + y_2 - y_1^2 z_2)}$$

$$\Gamma_o = \frac{j\cdot(z_2 - y_2 + y_1^2 z_2)}{2y_1z_2 + j(z_2 + y_2 - y_1^2 z_2)}$$

$$T_e = \frac{2}{-2y_1z_2 + j\cdot(z_2 + y_2 - y_1^2 z_2)}$$

$$T_o = \frac{2}{2y_1z_2 + j\cdot(z_2 + y_2 - y_1^2 z_2)}$$

$$b_1 = \frac{\Gamma_e + \Gamma_o}{2} = \frac{z_2^2 - (y_2 - y_1^2 z_2)^2}{(2y_1z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$b_2 = \frac{T_e + T_o}{2} = \frac{-2j(z_2 + y_2 - y_1^2 z_2)}{(2y_1z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$b_3 = \frac{T_e - T_o}{2} = \frac{-4y_1z_2}{(2y_1z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$b_4 = \frac{\Gamma_e - \Gamma_o}{2} = \frac{-2jy_1z_2(z_2 - y_2 + y_1^2 z_2)}{(2y_1z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$b_1 = 0 \Rightarrow z_2 - y_2 + y_1^2 z_2 = 0 \Rightarrow z_2^2 = \frac{1}{1 + y_1^2}$$

$$y_2^2 = 1 + y_1^2$$

$$C = 10\log\frac{P_1}{P_3} = -20\log|b_3|\,, dB$$

$$b_1 = 0 \quad b_4 = 0 \quad b_3 = -y_1z_2 \quad b_2 = -jz_2$$

$$b_3 = -\frac{\sqrt{y_2^2 - 1}}{y_2}\,,\; b_2 = -\frac{j}{y_2}$$

$$\beta = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$b_3 = -C$$

$$b_2 = -j\sqrt{1 - C^2}$$

$$[S] = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & -C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & -C \\ -C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & -C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

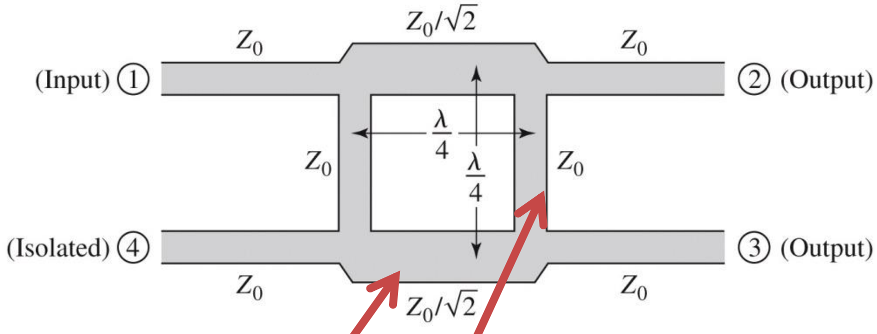


Figure 7.21

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

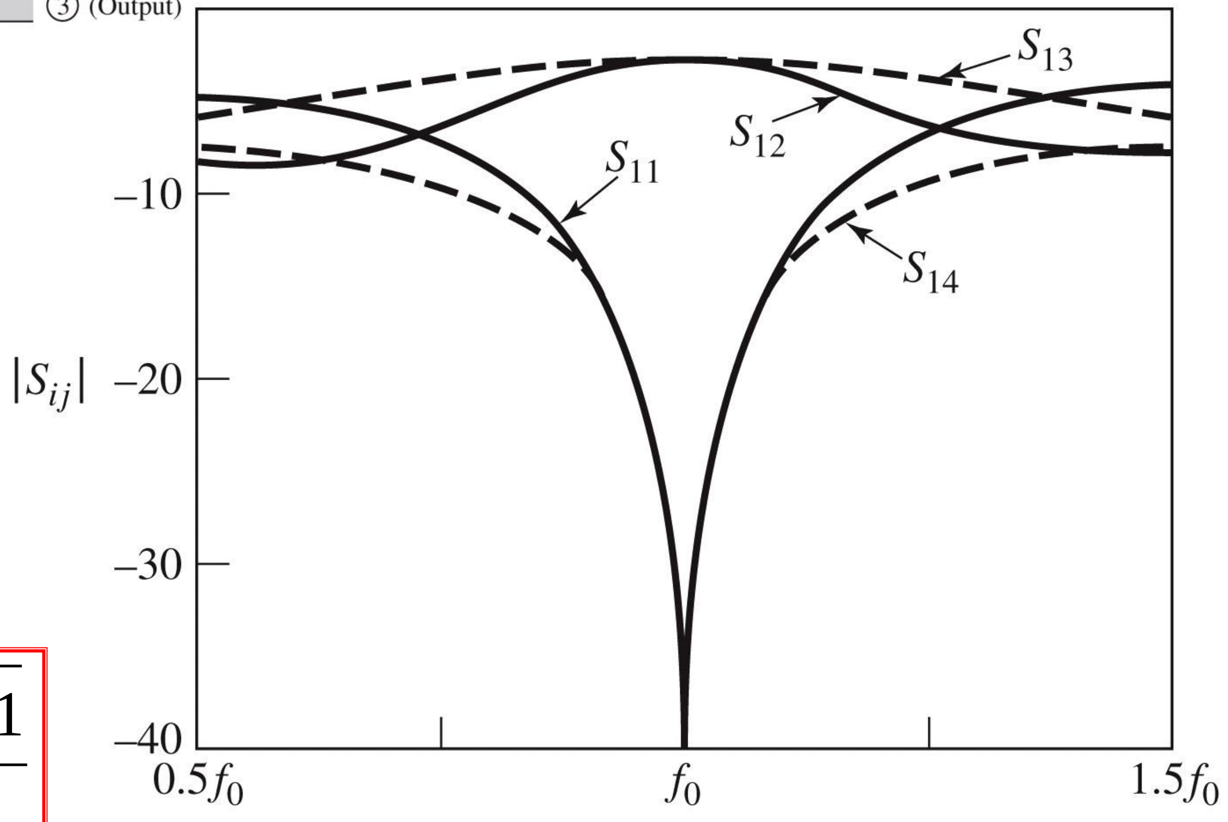


Figure 7.25

Example

Design a cuadrature (90°) hybrid working on 50 Ω, and plot the S parameters between

$0.5\,f_0$ and $1.5\,f_0$, where f_0

is the frequency at which the length of the branches is $\lambda/4$

.

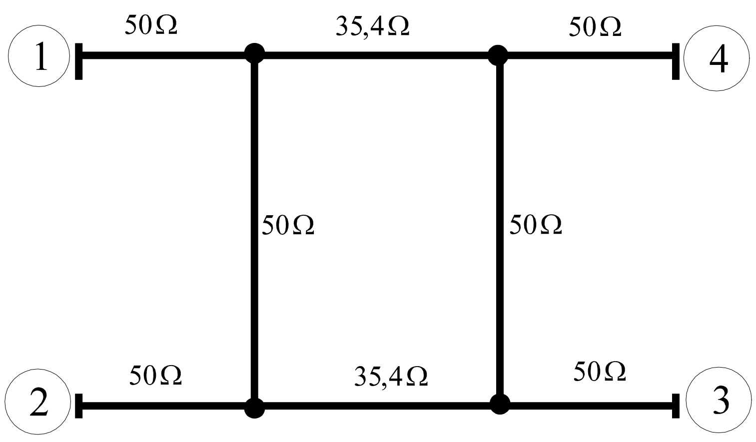
Solution

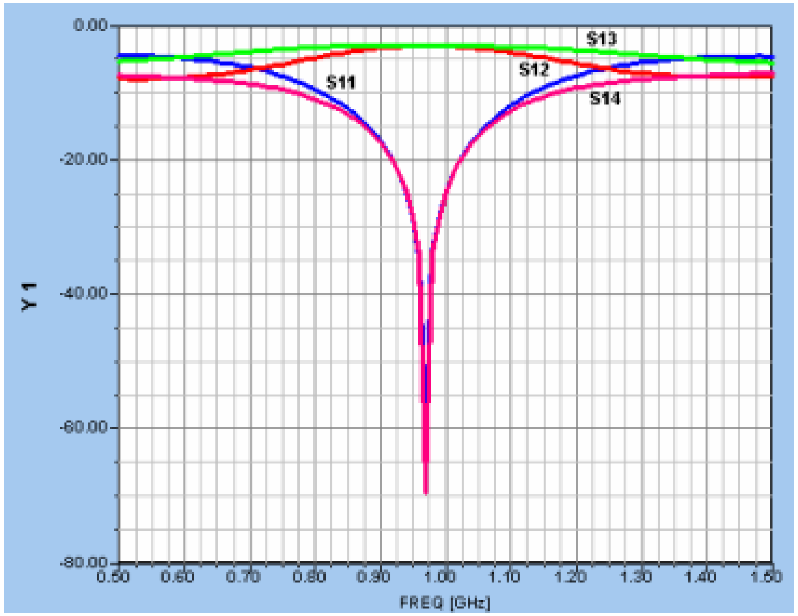
A cuadrature (90°) hybrid has C = 3dB, then $\beta = 1/\sqrt{2}$

$y_2 = \sqrt{2}$ and $y_1 = 1$

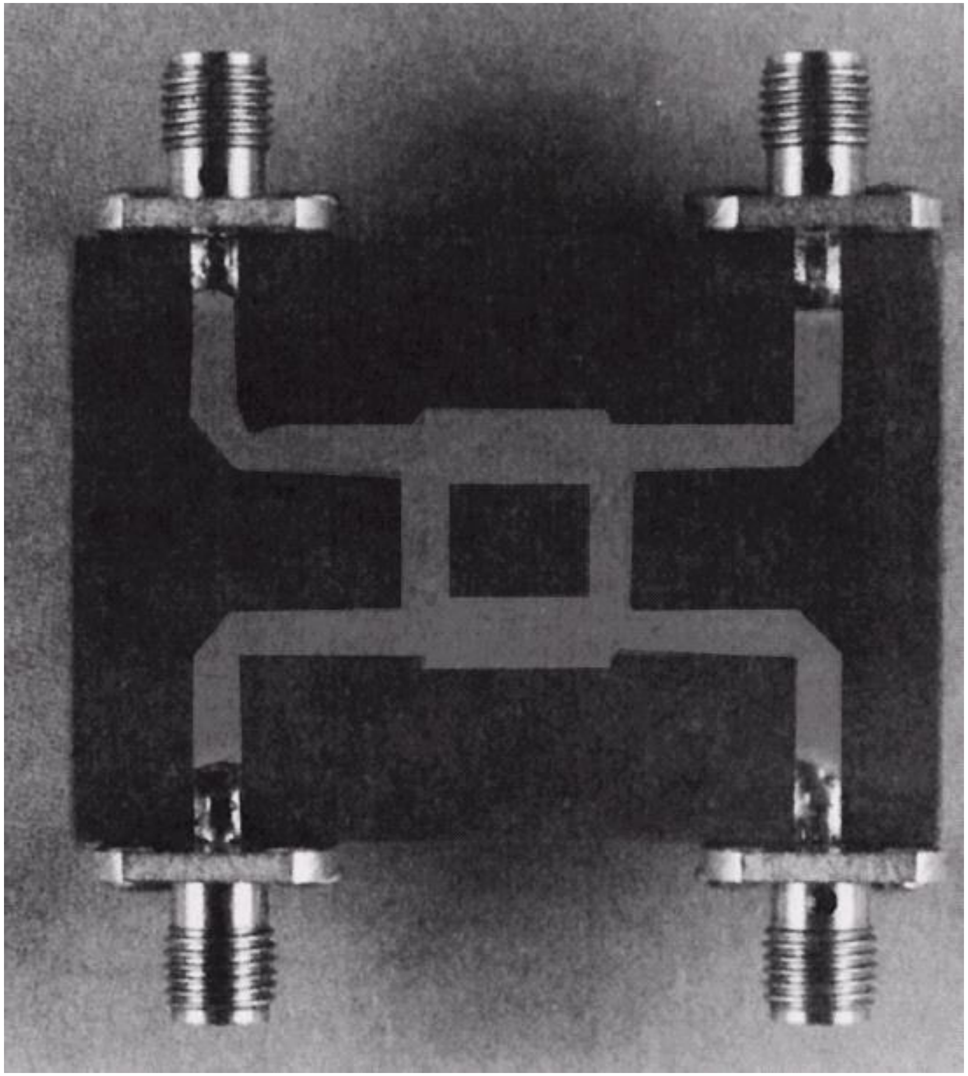
$Z_0 = 50\Omega$ the characteristic impedances will be:

$$Z_1 = Z_0 = 50\Omega \quad Z_2 = \frac{Z_0}{\sqrt{2}} = 35.4\Omega$$





The cuadrature (90°) hybrid



The cuadrature (90°) hybrid

- eight-way microstrip power divider with six quadrature hybrids in a Bailey configuration

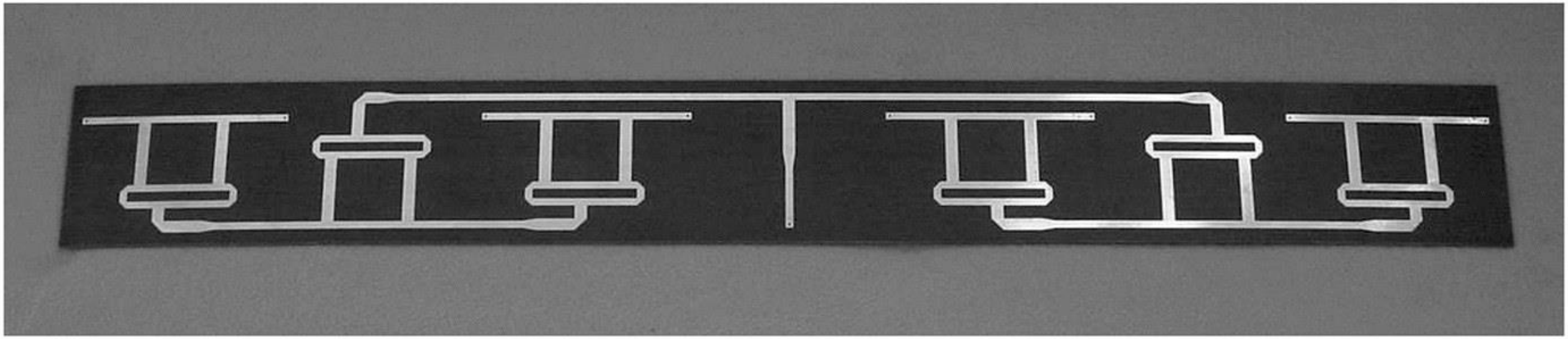
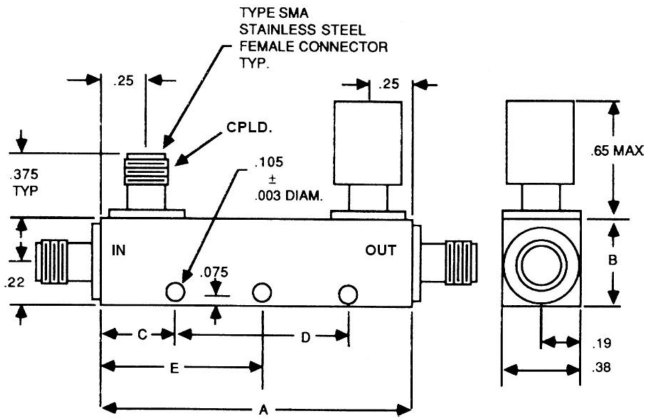


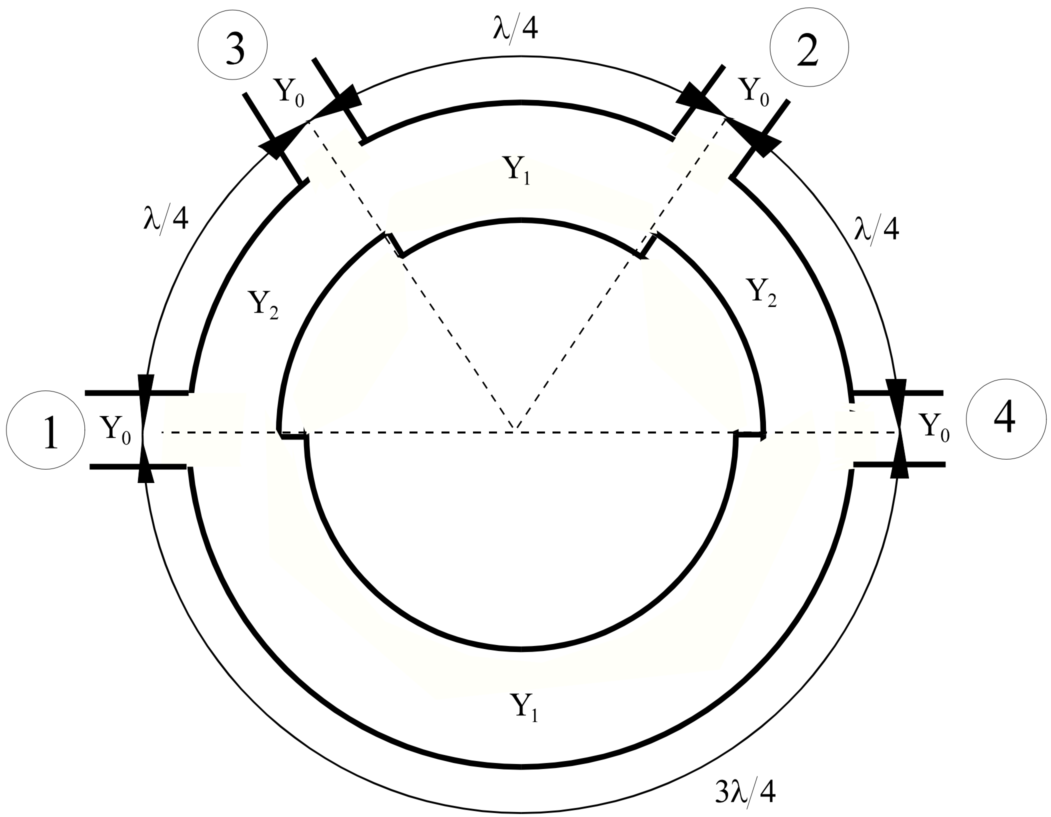
Figure 7.24
Courtesy of ProSensing, Inc., Amherst, Mass.

Datasheet



Model No.	Frequency Range (Ghz)	Coupling † (dB)	Freq. Sens. (dB)	Insertion Loss (dB)		Directivity (dB min.)	VSWR max.	
				Excl. Cpld Pwr	True		Primary Line	Secondary Line
MDC6223-6	0.5-1.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6223-10	0.5-1.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6223-20	0.5-1.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6223-30	0.5-1.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-6	1.0-2.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6224-10	1.0-2.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6224-20	1.0-2.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-30	1.0-2.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6225-6	2.0-4.0	6 ±1.00	±0.60	0.20	1.80	22	1.15	1.15
MDC6225-10	2.0-4.0	10 ±1.25	±0.75	0.20	0.80	22	1.15	1.15
MDC6225-20	2.0-4.0	20 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6225-30	2.0-4.0	30 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6266-6	2.6-5.2	6 ±1.00	±0.60	0.20	1.80	20	1.25	1.25
MDC6266-10	2.6-5.2	10 ±1.25	±0.75	0.20	0.80	20	1.25	1.25
MDC6266-20	2.6-5.2	20 ±1.25	±0.75	0.20	0.25	20	1.25	1.25
MDC6266-30	2.6-5.2	30 ±1.25	±0.75	0.20	0.20	20	1.25	1.25
MDC6226-6	4.0-8.0	6 ±1.00	±0.60	0.25	1.90	20	1.25	1.25
MDC6226-10	4.0-8.0	10 ±1.25	±0.75	0.25	0.90	20	1.25	1.25
MDC6226-20	4.0-8.0	20 ±1.25	±0.75	0.25	0.30	20	1.25	1.25
MDC6226-30	4.0-8.0	30 ±1.25	±0.75	0.25	0.25	20	1.25	1.25
MDC6227-6	7.0-12.4	6 ±1.00	±0.50	0.30	2.00	17	1.30	1.30
MDC6227-10	7.0-12.4	10 ±1.00	±0.50	0.30	1.00	17	1.30	1.30
MDC6227-20	7.0-12.4	20 ±1.00	±0.50	0.30	0.35	17	1.30	1.30

The 180° ring hybrid (rat-race)



The 180° ring hybrid

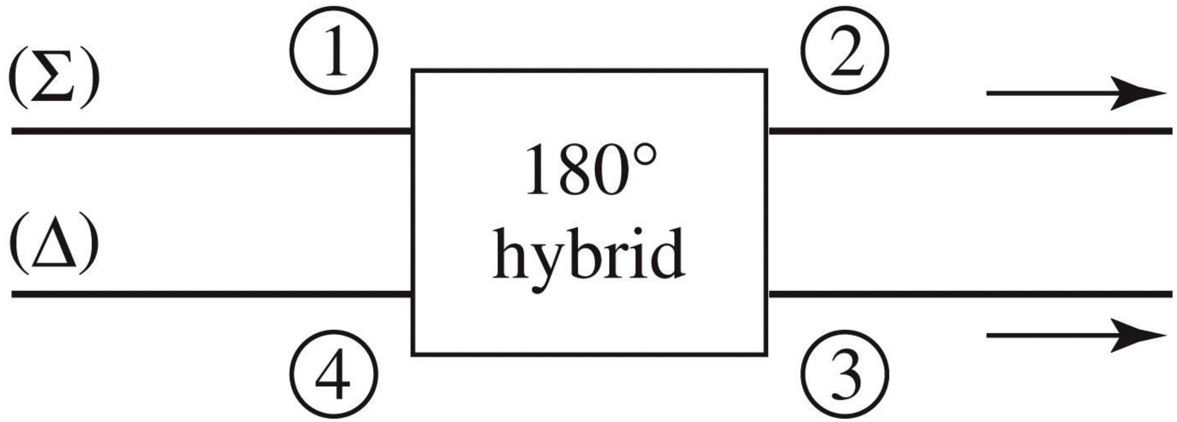
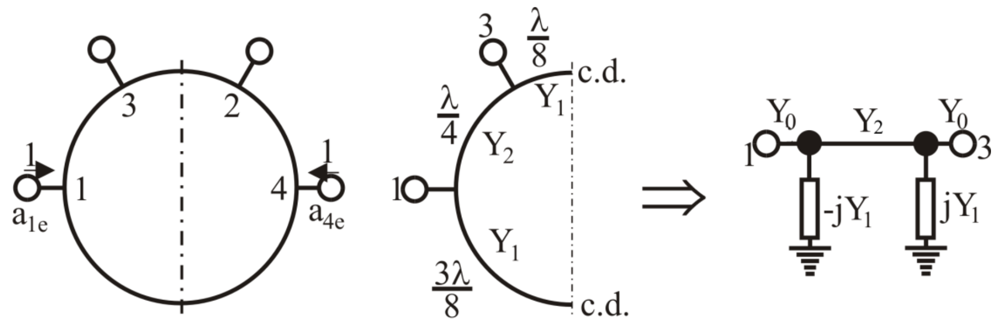


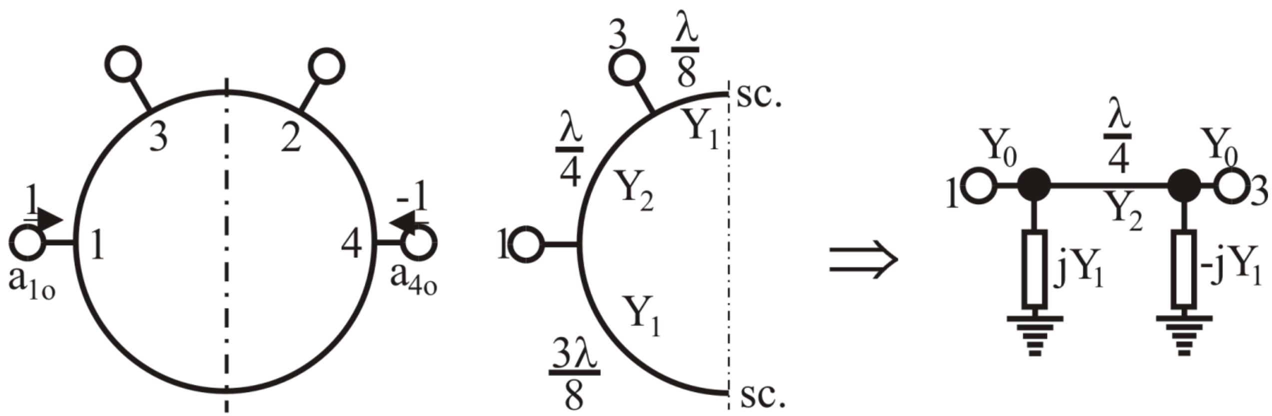
Figure 7.41

- The 180° ring hybrid can be operated in different modes:
  - a signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3
  - input applied to port 4 it will be equally split into two components with a 180◦ phase difference at ports 2 and 3
  - input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4 (power combiner)

Even/Odd Mode Analysis



Even Mode

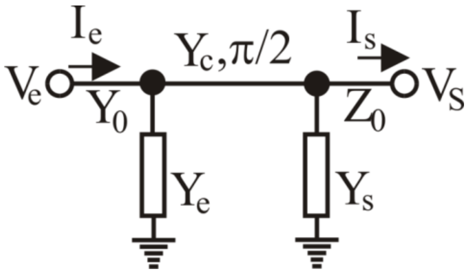


Odd Mode

Even/Odd Mode Analysis

$$S_{11}=\frac{jz_2y_s+jz_2-j(y_2+y_ey_sz_2)-jy_ez_2}{jz_2y_s+jz_2+j(y_2+y_ey_sz_2)+jy_ez_2}$$

$$S_{12}=\frac{2}{jz_2y_s+jz_2+j(y_2+y_ey_sz_2)+jy_ez_2}$$



$$S_{21}=\frac{2}{jz_2y_s+jz_2+j(y_2+y_ey_sz_2)+jy_ez_2}$$

$$S_{22}=\frac{-jz_2y_s+jz_2-j(y_2+y_ey_sz_2)+jy_ez_2}{jz_2y_s+jz_2+j(y_2+y_ey_sz_2)+jy_ez_2}$$

Even mode:

$$y_e=-jy_1$$
$$y_s=jy_1$$

Matching condition

$$y_1^2+y_2^2=1$$

Odd mode:

$$y_e=jy_1$$
$$y_s=-jy_1$$

$$S_{11e}=\frac{z_2-y_2-y_1^2z_2+2jz_2y_1}{z_2+y_2+y_1^2z_2}$$

$$S_{12e}=S_{21e}=\frac{-2j}{z_2+y_2+y_1^2z_2}$$

$$S_{22e}=\frac{z_2-y_2-y_1^2z_2-2jz_2y_1}{z_2+y_2+y_1^2z_2}$$

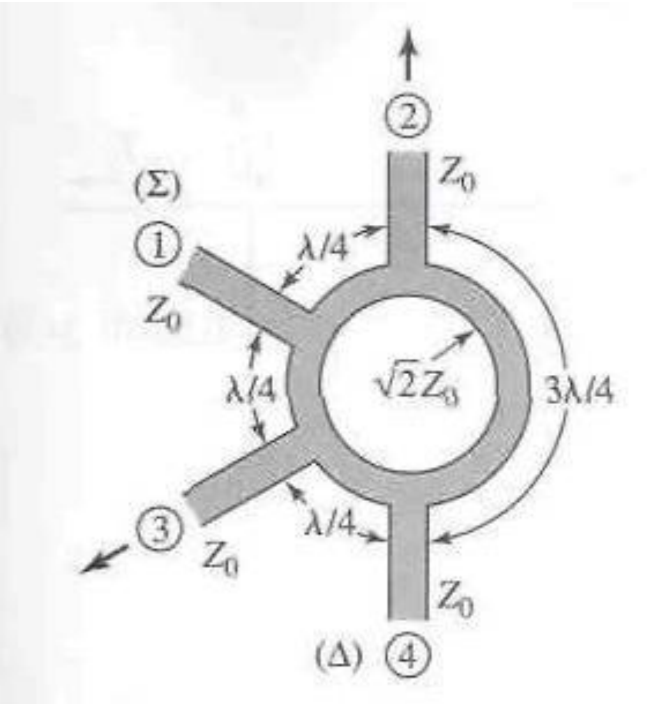
$$[S]=\begin{bmatrix}0 & 0 & -jy_2 & jy_1\\ 0 & 0 & -jy_1 & -jy_2\\ -jy_2 & -jy_1 & 0 & 0\\ jy_1 & -jy_2 & 0 & 0\end{bmatrix}$$

$$S_{11o}=\frac{z_2-y_2-y_1^2z_2-2jz_2y_1}{z_2+y_2+y_1^2z_2}$$

$$S_{12o}=S_{21o}=\frac{-2j}{z_2+y_2+y_1^2z_2}$$

$$S_{22o}=\frac{z_2-y_2-y_1^2z_2+2jz_2y_1}{z_2+y_2+y_1^2z_2}$$

The 180° ring hybrid



$$[S]=\begin{bmatrix} 0 & -jy_2 & -jy_1 & 0 \\ -jy_2 & 0 & 0 & jy_1 \\ -jy_1 & 0 & 0 & -jy_2 \\ 0 & jy_1 & -jy_2 & 0 \end{bmatrix} = -j\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$C(dB) = -20\log(\beta) = -20\log(y_1)$$

Example

Design a ring (180°) hybrid working on 50 Ω, and plot the S parameters between 0.5 and 1.5 of the design frequency.

$$C\,[\mathrm{dB}] = -20\log(y_1)$$

$$\sqrt{2}Z_0 = 70.7\Omega$$

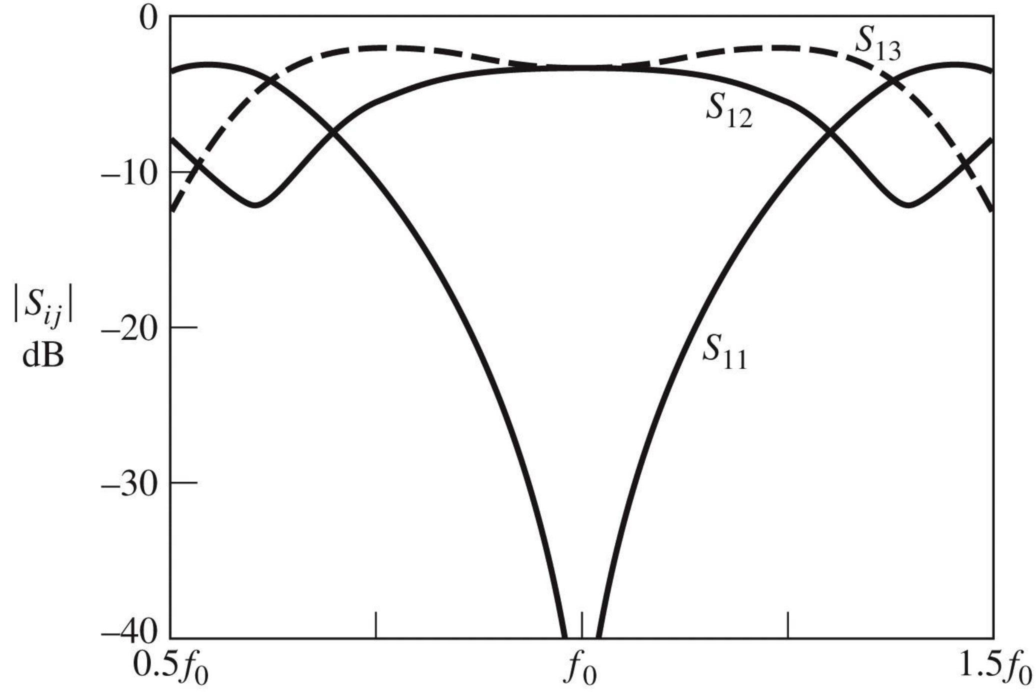
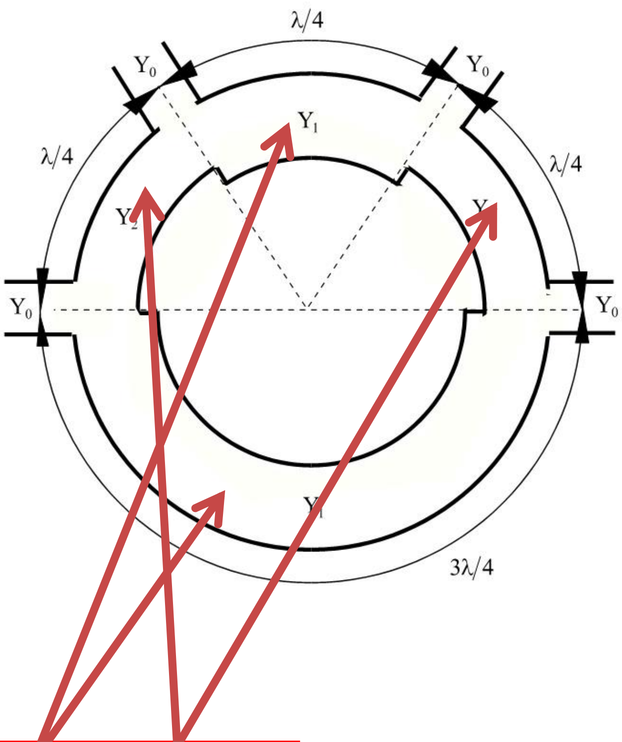


Figure 7.46

The 180° ring hybrid



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}(y_1)$$

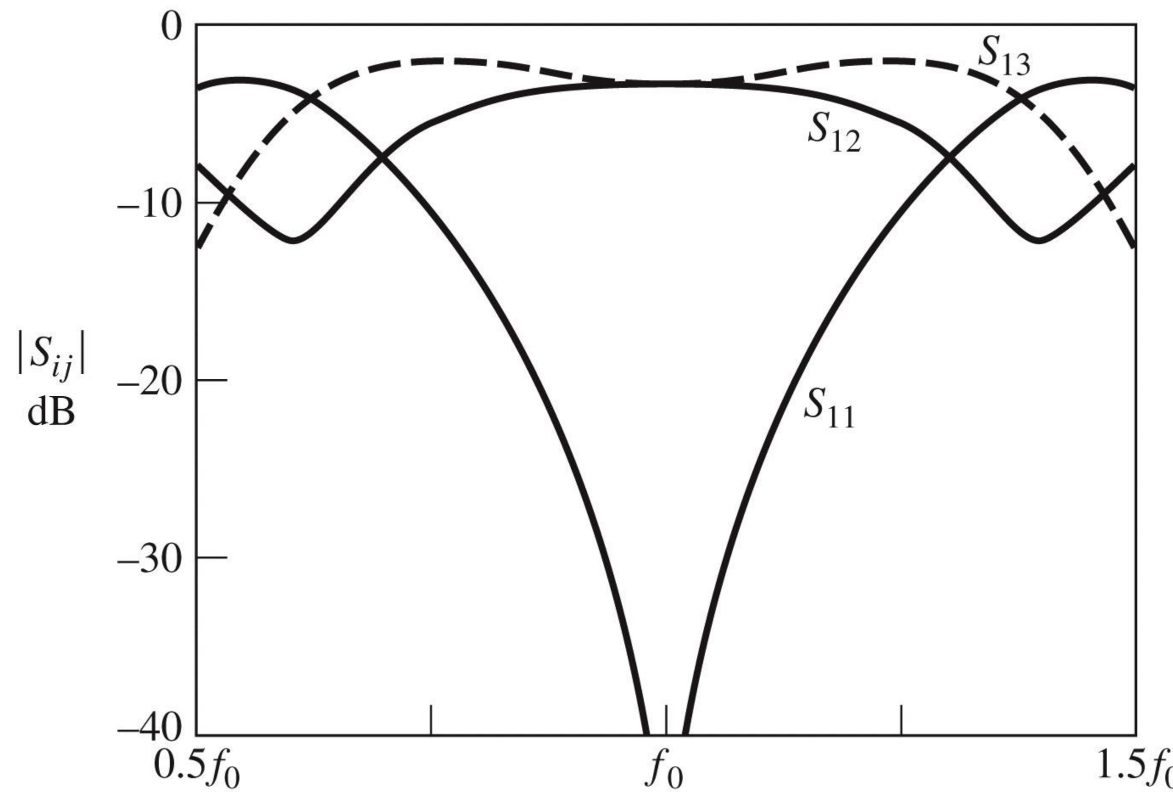


Figure 7.46

The 180° ring hybrid

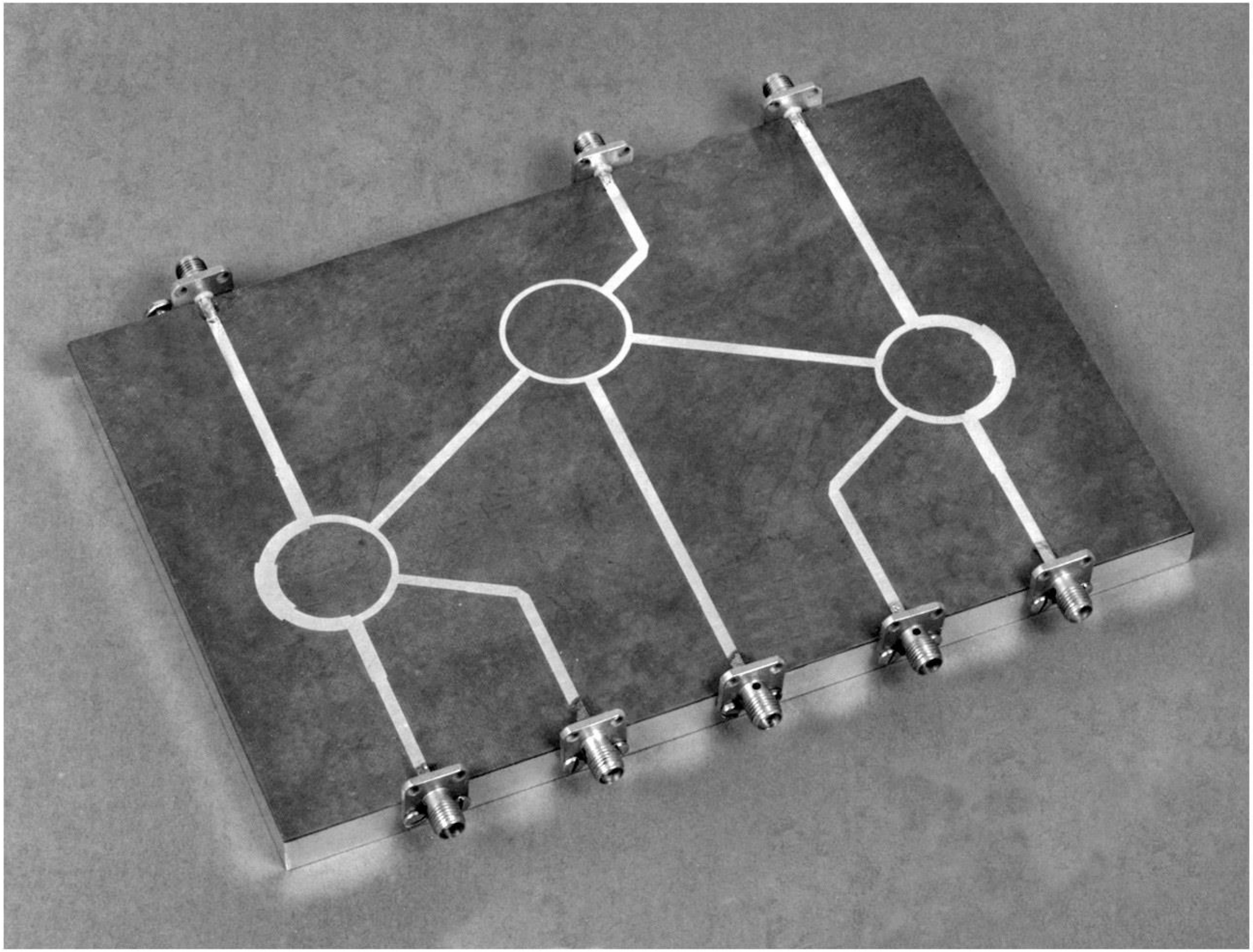
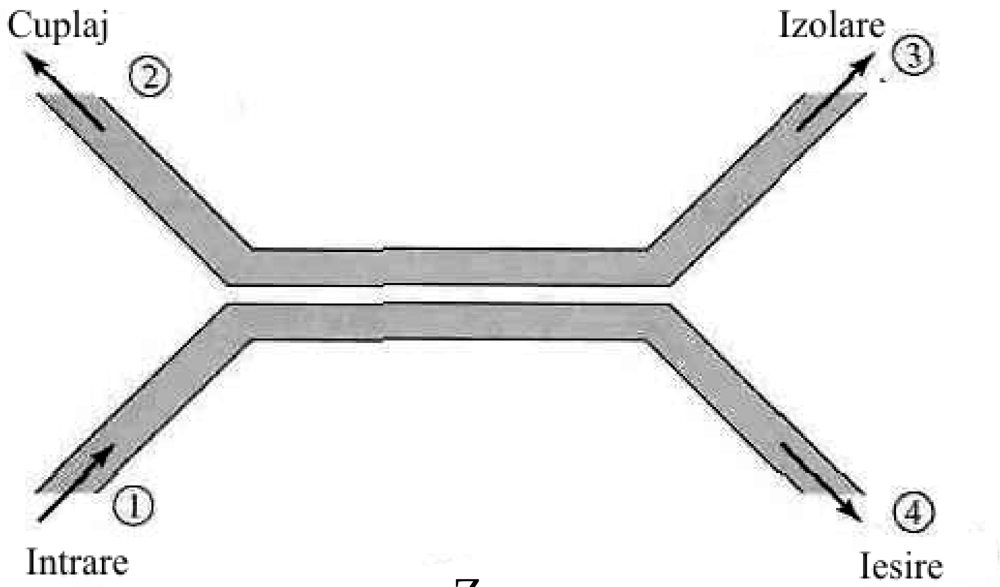
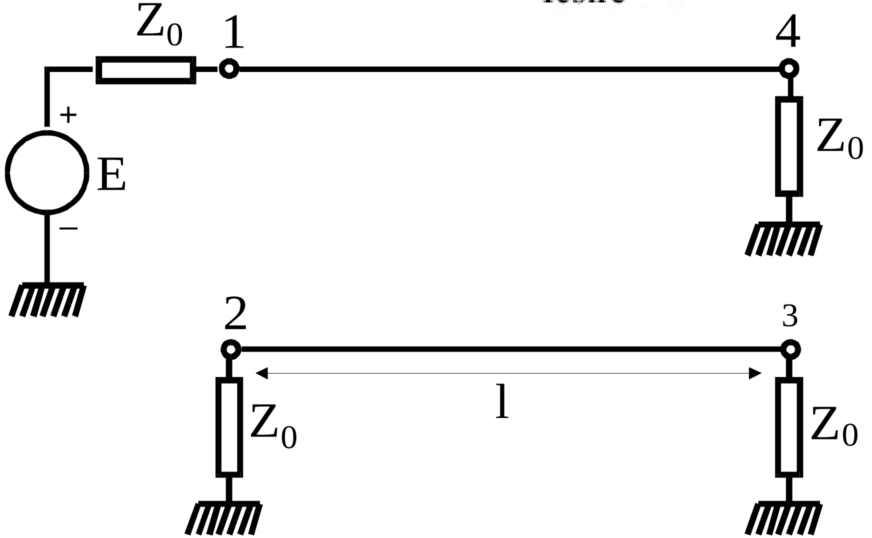


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler





Coupled Lines

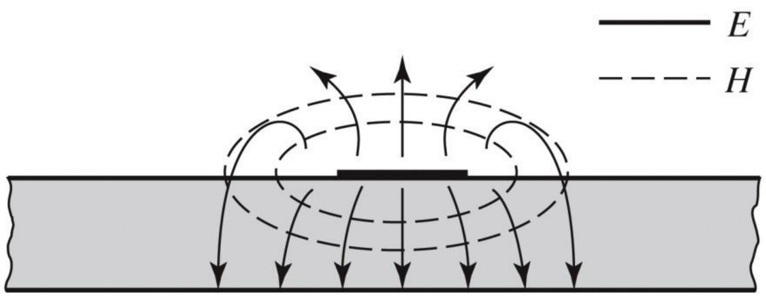
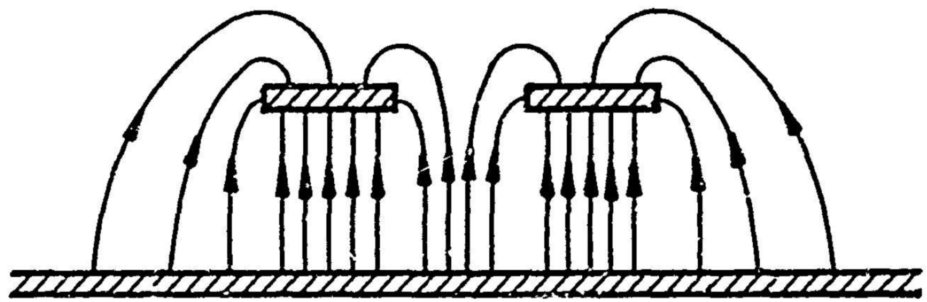
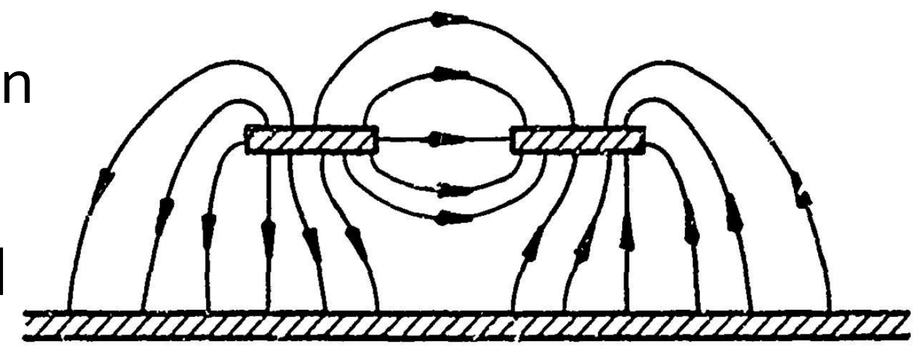


Figure 3.25b



b) EVEN MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

- Even mode - characterizes the common mode signal on the two lines
- Odd mode - characterizes the differential mode signal between the two lines
- Each of the two modes is characterized by **different** characteristic impedances



c) ODD MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

Coupled Lines

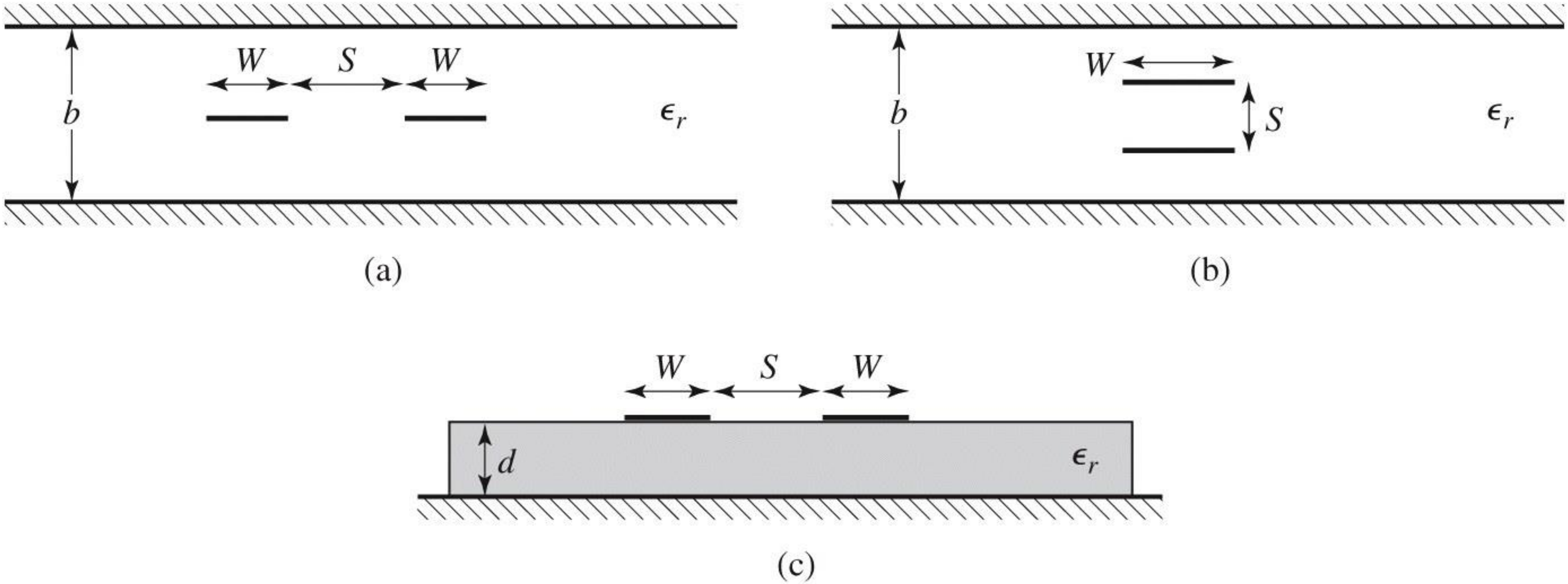


Figure 7.26

Coupled Lines

Figure 7.27

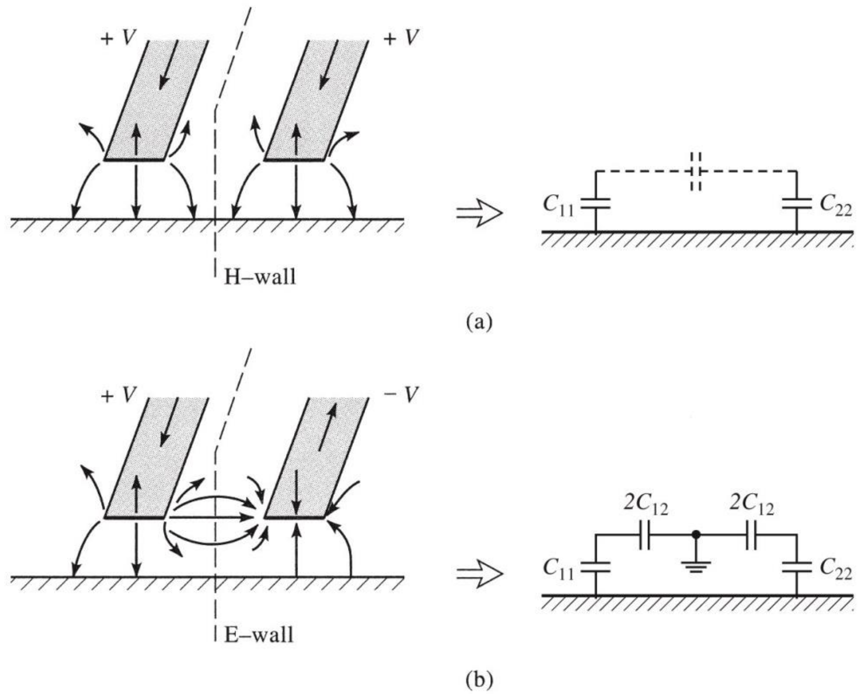
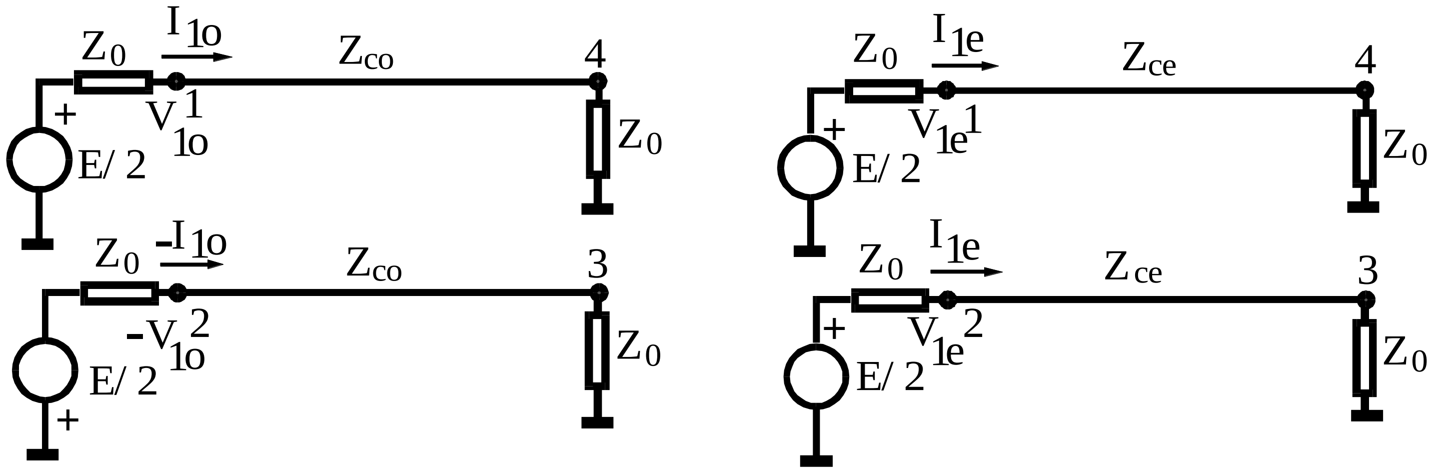


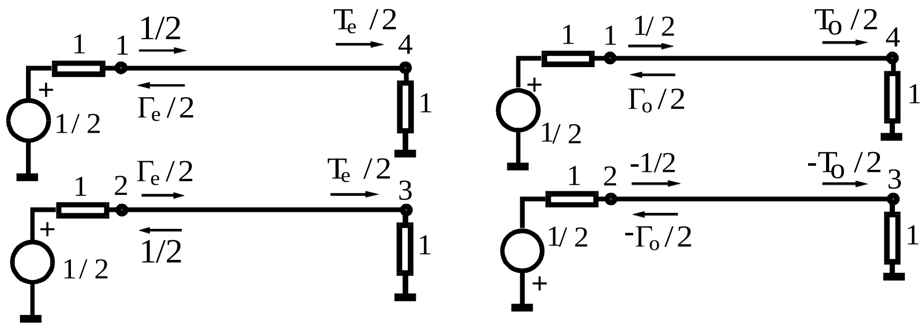
Figure 7.28

Matching in Coupled Line Coupler



$$\begin{cases} Z_{ce} Z_{co} = Z_0^2 \\ \theta_e = \theta_o \end{cases}$$

Directivity and Coupling factor



modul par

modul impar

$$a_1 = a_{1e} + a_{1o} = 1\,,\; a_2 = a_3 = a_4 = 0$$

$$b_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 \Leftrightarrow$$

$$b_2 = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{jC\sin(\theta)}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$b_3 = \frac{1}{2}(T_e - T_o) = 0$$

$$b_4 = \frac{1}{2}(T_e + T_o) = \frac{\sqrt{1-C^2}}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$C = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\theta = \pi/2$$

$$[S] = \begin{bmatrix} 0 & C & 0 & -j\sqrt{1-C^2} \\ C & 0 & -j\sqrt{1-C^2} & 0 \\ 0 & -j\sqrt{1-C^2} & 0 & C \\ -j\sqrt{1-C^2} & 0 & C & 0 \end{bmatrix}$$

Coupled Line Coupler

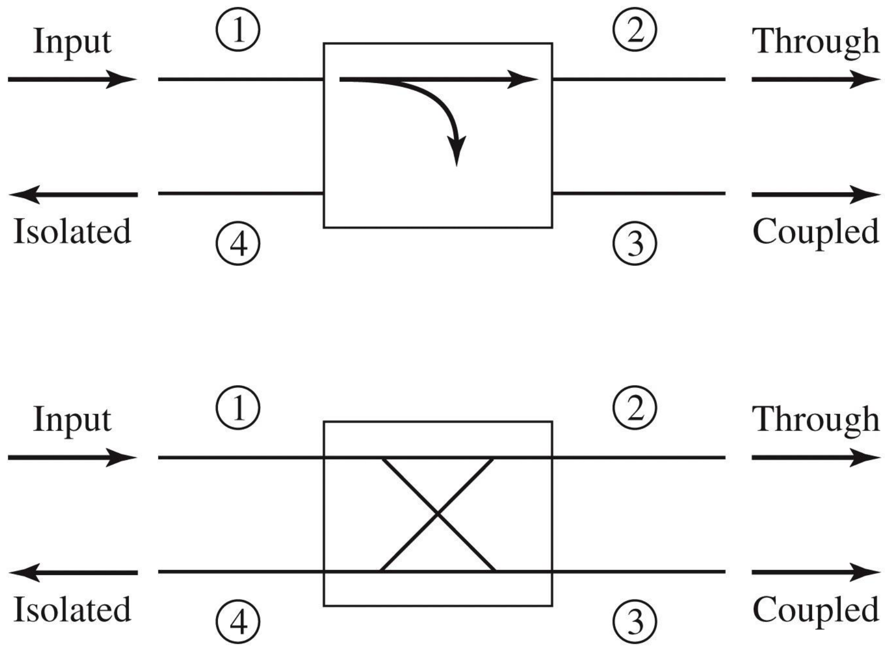
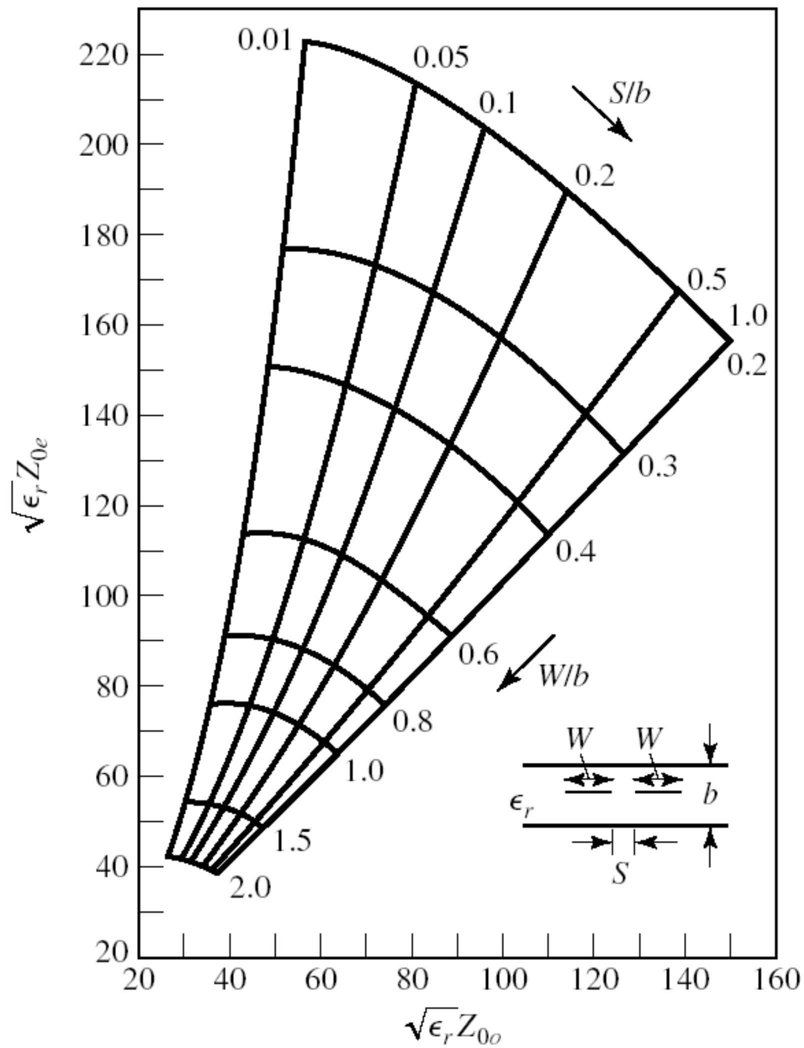


Figure 7.4

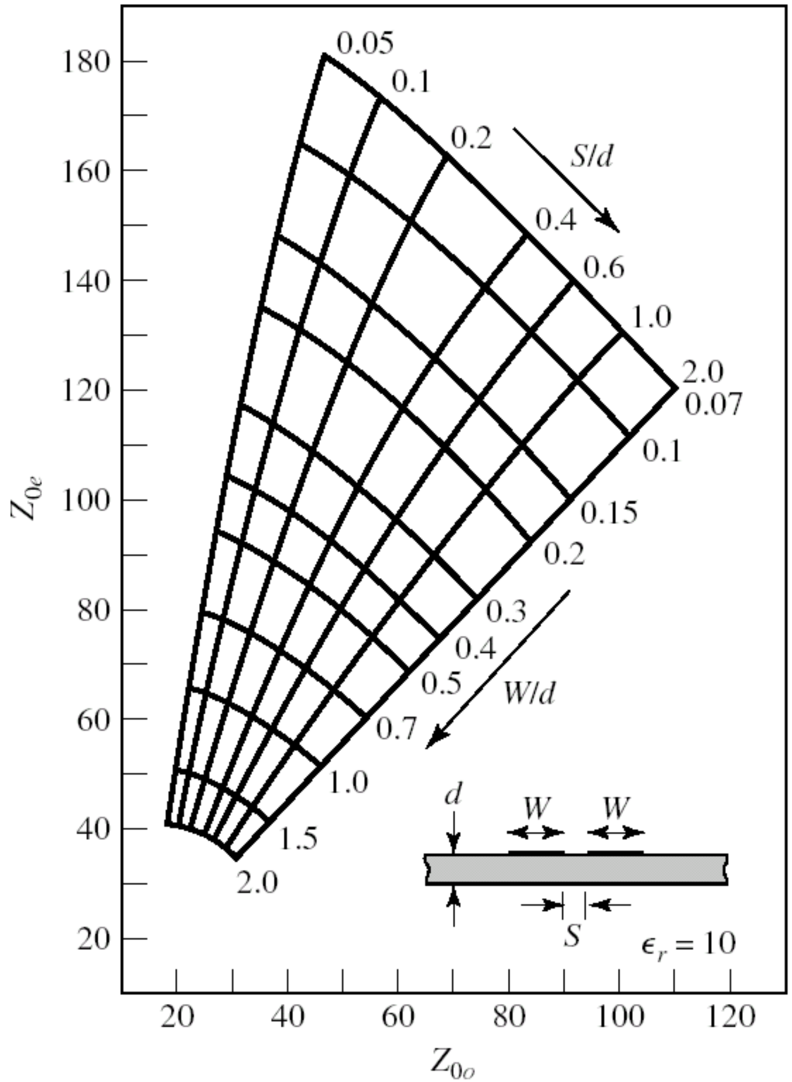
$$[S] = -j \cdot \begin{bmatrix} 0 & \sqrt{1-C^2} & jC & 0 \\ \sqrt{1-C^2} & 0 & 0 & jC \\ jC & 0 & 0 & \sqrt{1-C^2} \\ 0 & jC & \sqrt{1-C^2} & 0 \end{bmatrix}$$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

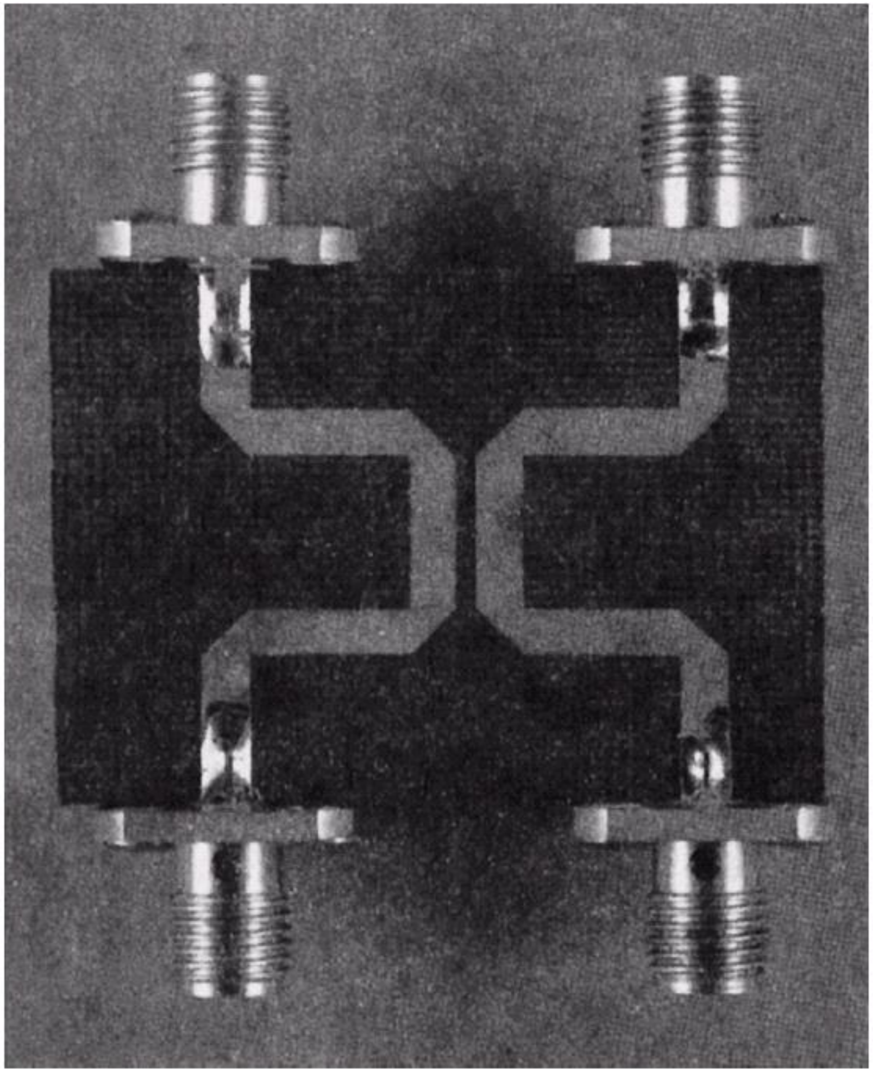
Normalized even- and odd-mode characteristic impedance design data for edge-coupled striplines.



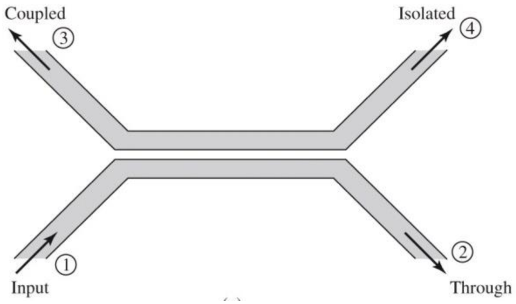
Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with $\varepsilon_r = 10$.



Coupled Line Coupler



Coupled Line Coupler



$$Z_{ce}Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

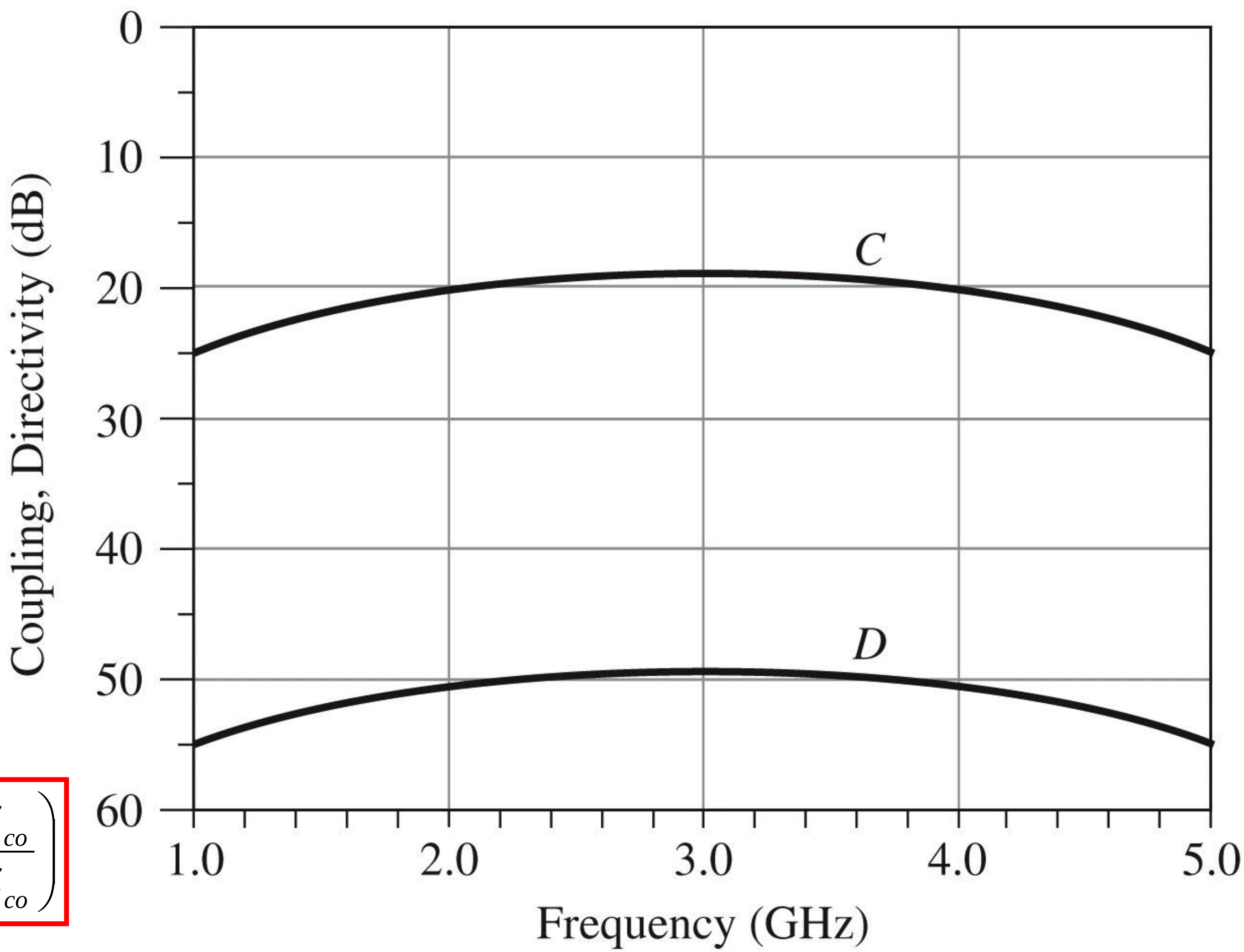


Figure 7.34

Example

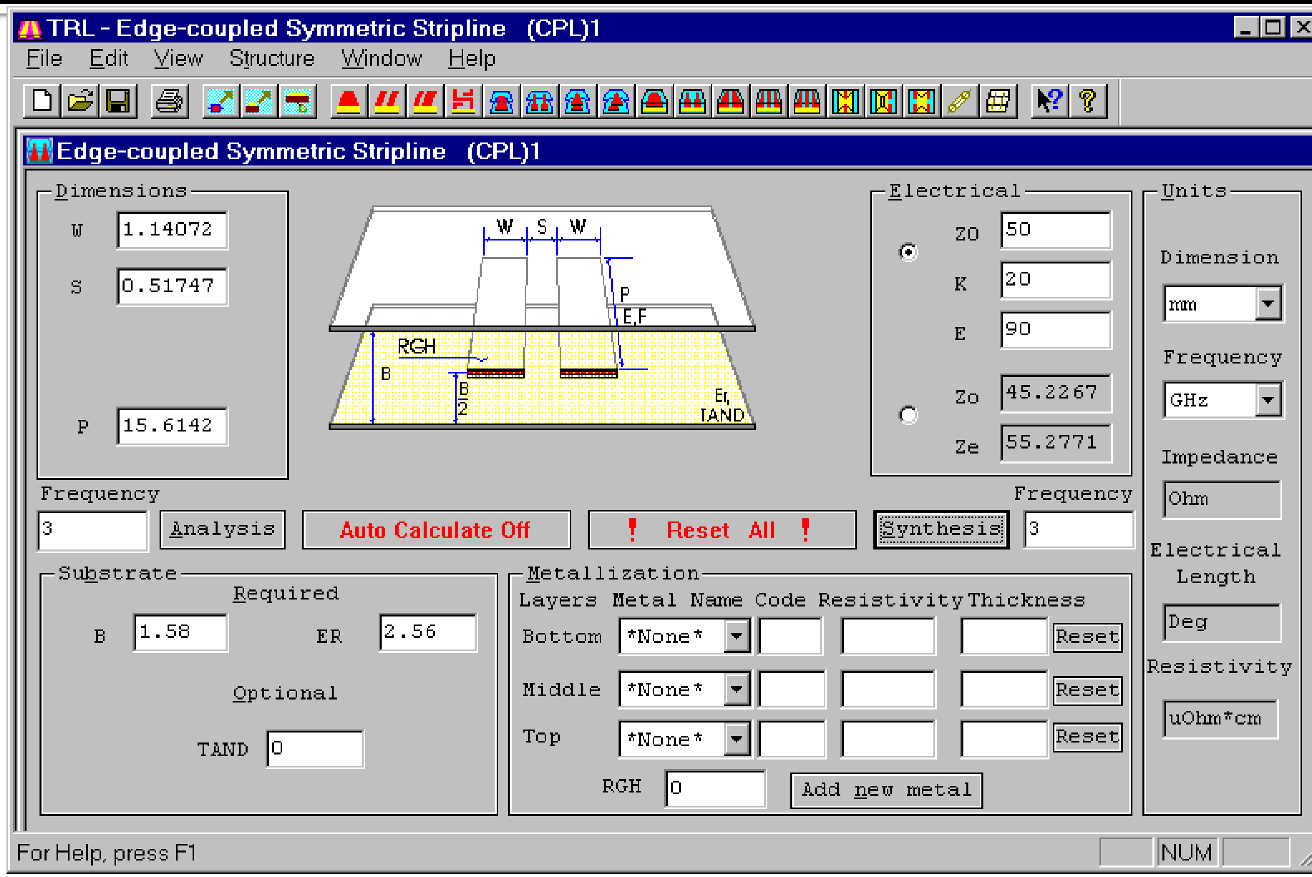
Design a coupled line coupler with 20 dB coupling factor, using stripline technology, with a distance between ground planes of 0.158 cm and an electrical permittivity of 2.56, working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz.

Solution

$$C = 10^{-20/20} = 0.1$$

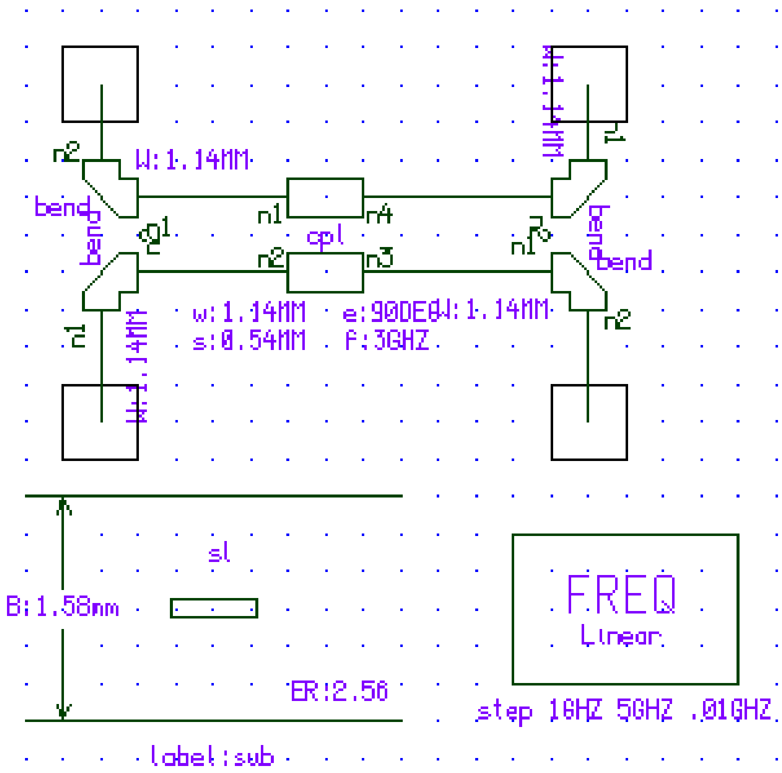
$$Z_{co} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\Omega$$

$$Z_{ce} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\Omega$$

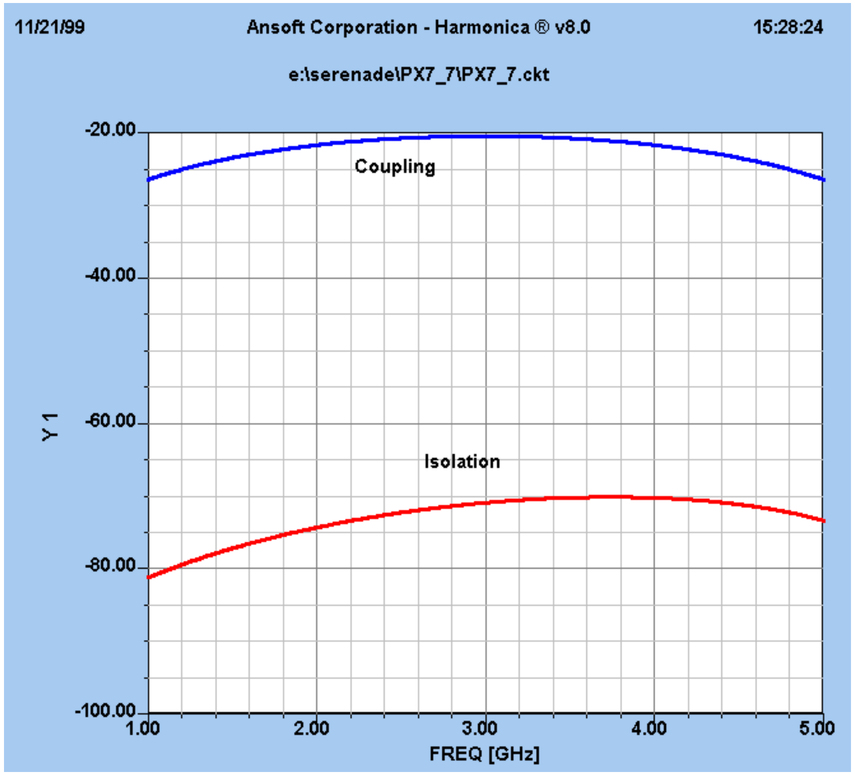


$$Z_{ce} = Z_0\sqrt{\frac{1+C}{1-C}}, \quad Z_{co} = Z_0\sqrt{\frac{1-C}{1+C}}$$

Simulation



W:1.14MM
n2
bend
n1
n4
cpl
n2
n3
n1
bend
w:1.14MM
s:0.54MM
e:90DEG
f:3GHZ
W:1.14MM
n2
n1
sl
B:1.58mm
ER:2.56
label:sub
FREQ
Linear
step 1GHZ 5GHZ .01GHZ



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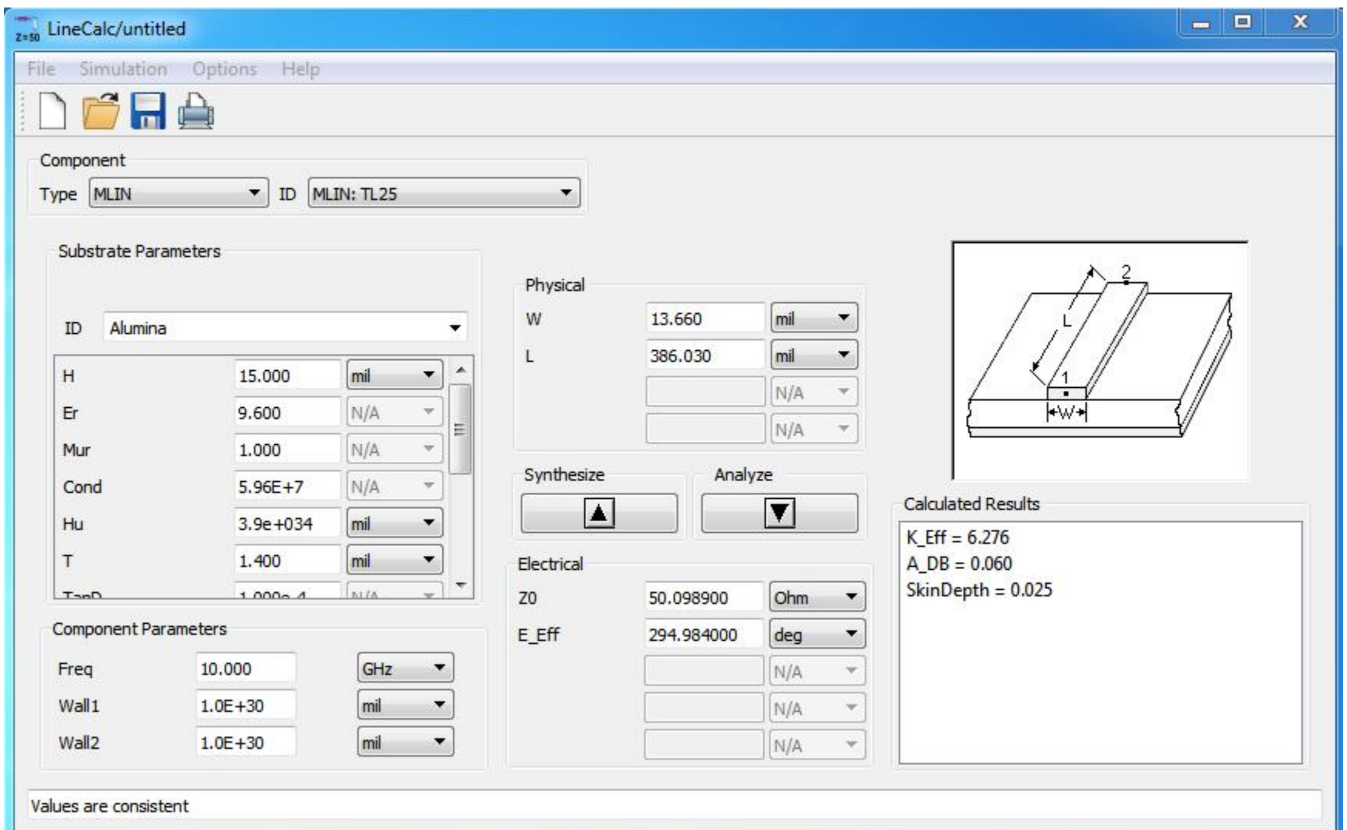
15:28:24
e:\serenade\PX7_7\PX7_7.ckt
Coupling
Isolation
Y 1
-20.00
-40.00
-60.00
-80.00
-100.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

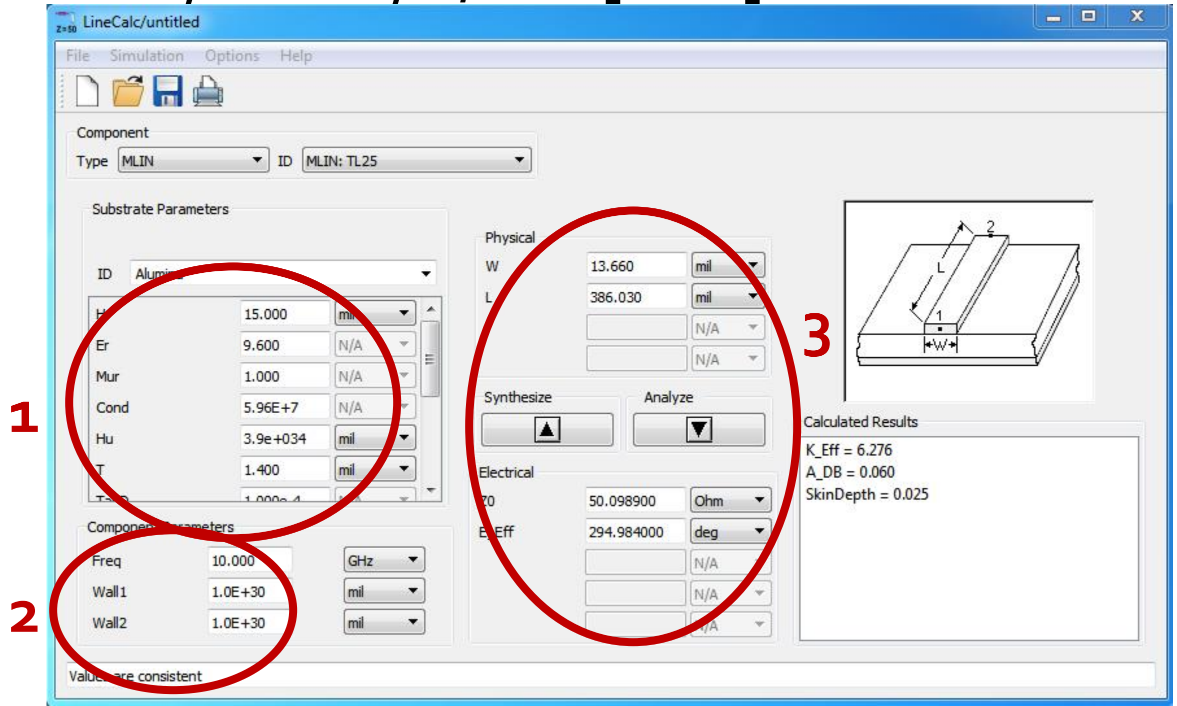
ADS linecalc

- In schematics: >Tools>LineCalc>Start
- for Microstrip lines >Tools>LineCalc>Send to Linecalc



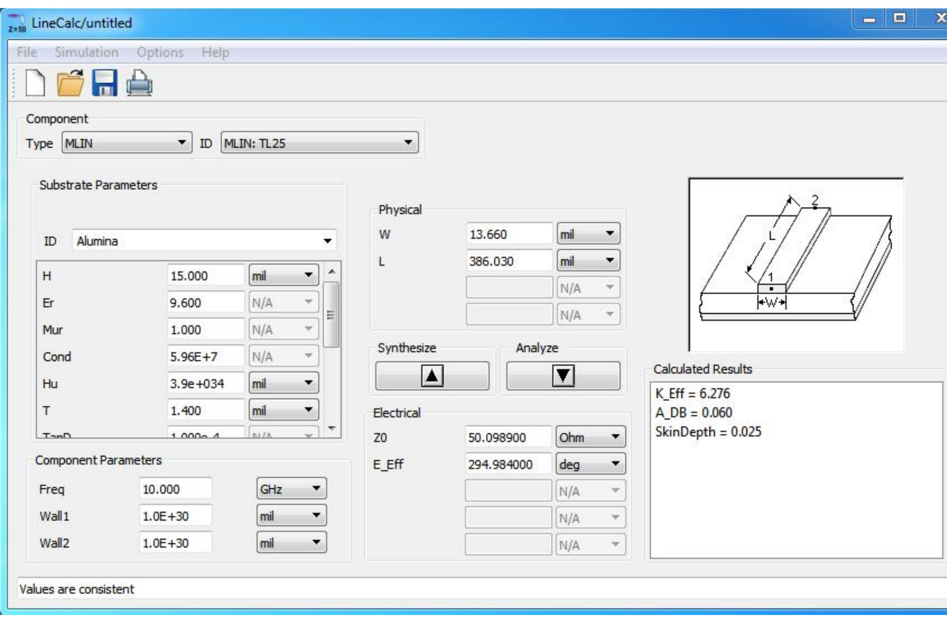
ADS linecalc

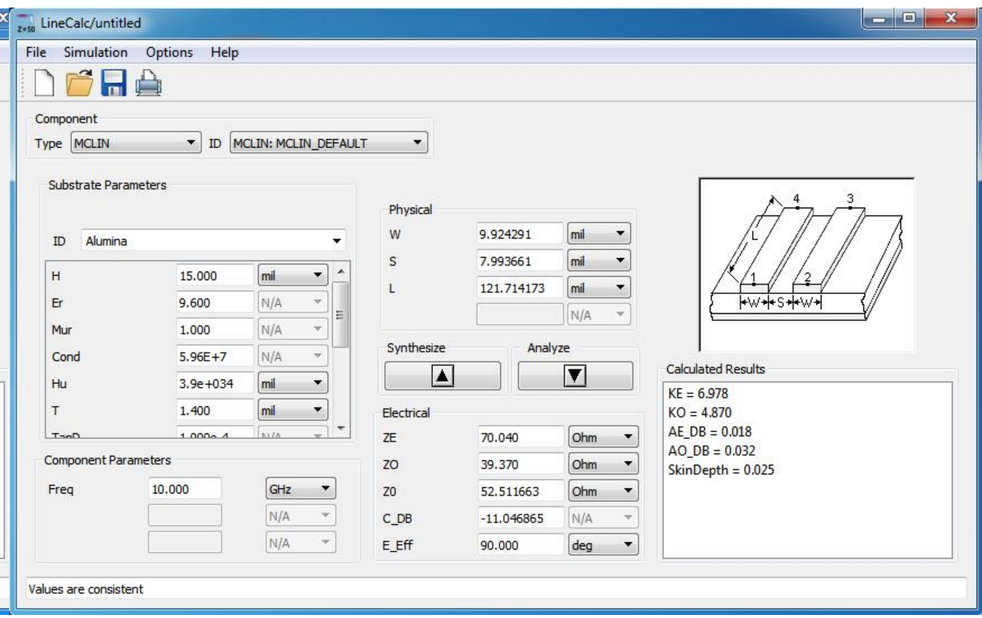
- 1. Define substrate (receive from schematic)
- 2. Insert frequency
- 3. Insert input data
  - Analyze: W,L → Zo,E or Ze,Zo,E / at f [GHz]
  - Synthesis: Zo,E → W,L / at f [GHz]



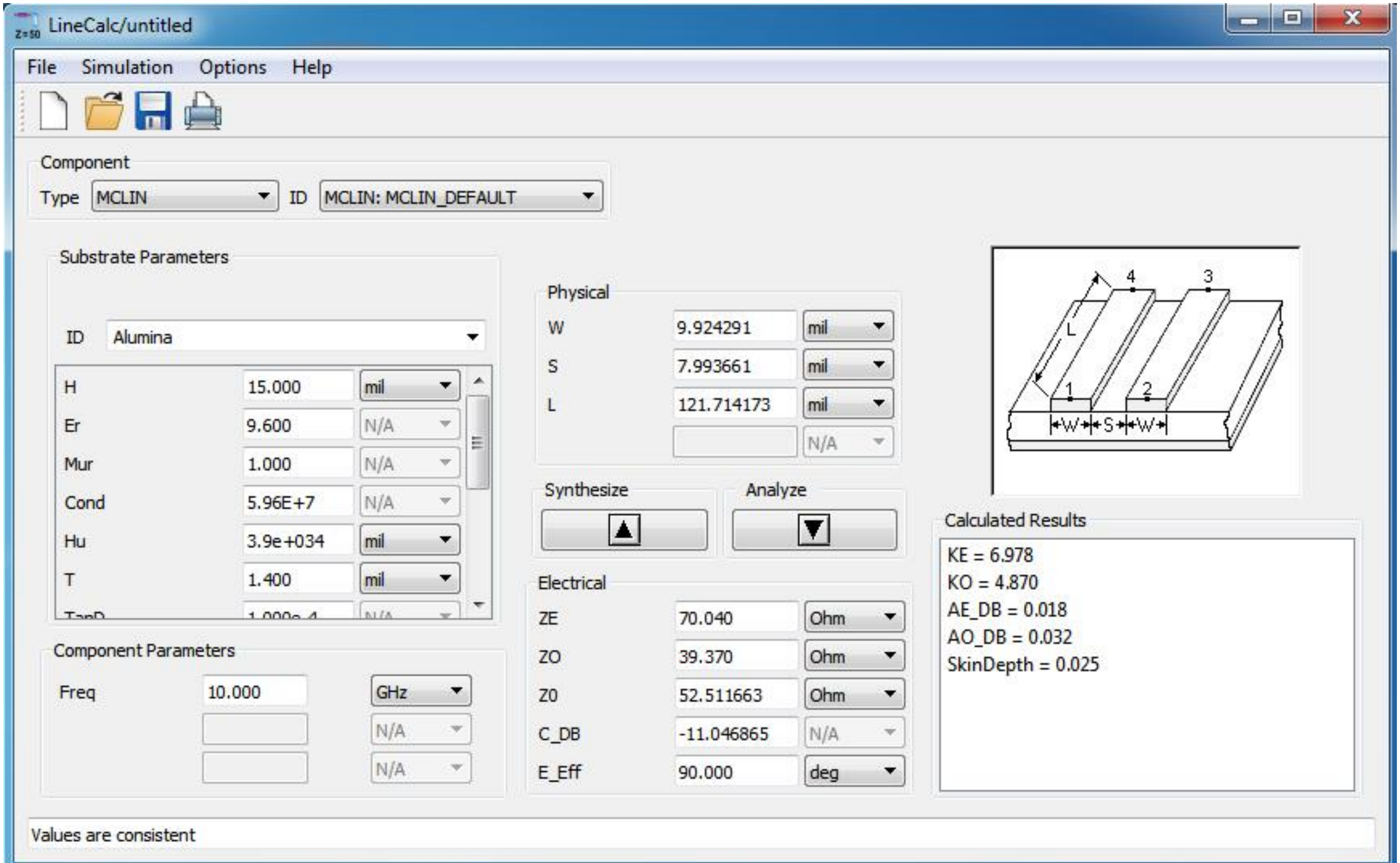
ADS linecalc

- Can be used for:
  - microstrip lines MLIN: W,L ⇔ Zo,E
  - microstrip coupled lines MCLIN: W,L,S ⇔ Ze,Zo,E

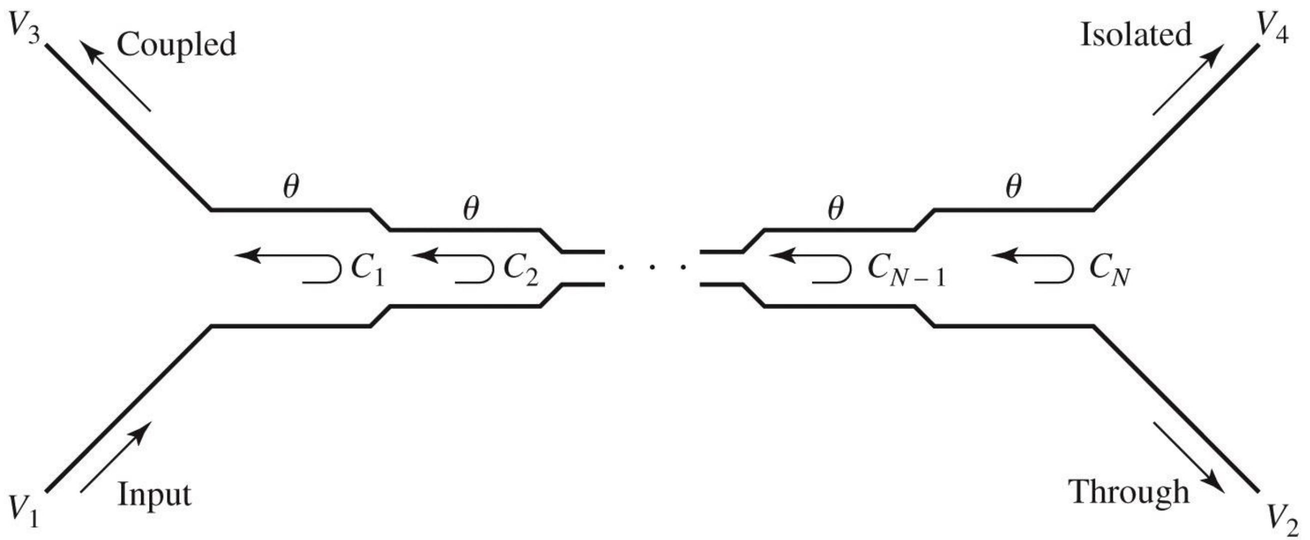




ADS linecalc



Multisection Coupled Line Couplers



C<<1

Figure 7.35

$$\frac{V_3}{V_1} = b_3 = \frac{jC\sin\theta}{\cos\theta\sqrt{1-C^2} + j\sin\theta} = \frac{jCtg\theta}{\sqrt{1-C^2} + jtg\theta} \approx \frac{jCtg\theta}{1+jtg\theta} = jC\sin\theta e^{-j\theta}$$

$$\frac{V_2}{V_1} = b_2 = \frac{\sqrt{1-C^2}}{\cos\theta\sqrt{1-C^2} + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin} = e^{-j\theta}$$

$$C = \frac{V_3}{V_1} = 2j\sin\theta e^{-j\theta}e^{-j(N-1)\theta}\left[C_1\cos(N-1)\theta + C_2\cos(N-3)\theta + \ldots + \frac{1}{2}C_{\frac{N+1}{2}}\right]$$

Example

Design a three sections coupled line coupler with 20 dB coupling factor, binomial characteristic (maximum flat), working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz

Solution

$$\left.\frac{d^n}{d\theta^n}C(\theta)\right|_{\theta=\pi/2} = 0,\ n = 1,2$$

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \frac{1}{2}C_2\right] = C_1(\sin 3\theta - \sin\theta) + C_2\sin\theta$$

$$\frac{dC}{d\theta} = \left[3C_1\cos 3\theta + (C_2 - C_1)\cos\theta\right]\Big|_{\theta=\pi/2} = 0$$

$$\frac{d^2C}{d\theta^2} = \left[-9C_1\sin 3\theta - (C_2 - C_1)\sin\theta\right]\Big|_{\theta=\pi/2} = 10C_1 - C_2 = 0$$

$$\begin{cases} C_2 - 2C_1 = 0.1 \\ 10C_1 - C_2 = 0 \end{cases}$$

$$\begin{cases} C_1 = C_3 = 0.0125 \\ C_2 = 0.125 \end{cases}$$

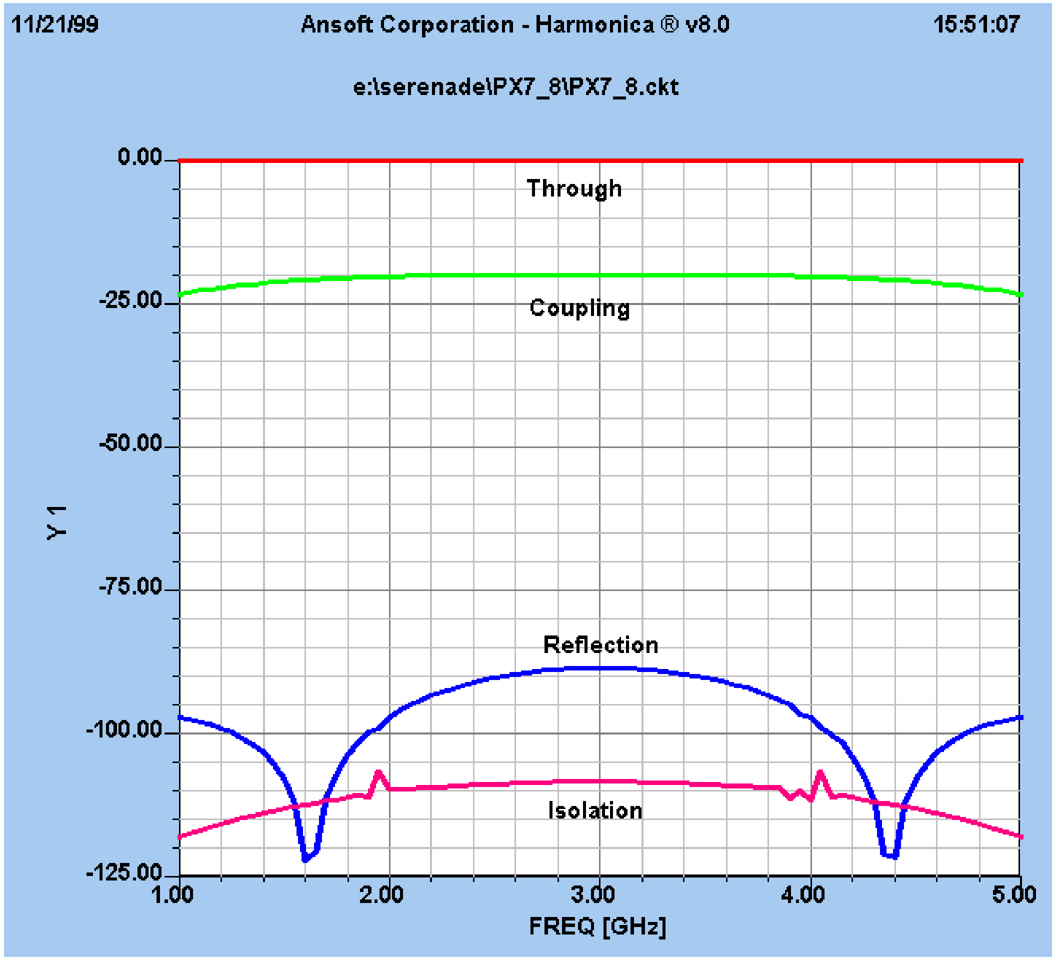
$$Z_{0e}^1 = Z_{0e}^3 = 50\sqrt{\frac{1.0125}{0.9875}} = 50.63\,\Omega$$

$$Z_{0o}^1 = Z_{0o}^3 = 50\sqrt{\frac{0.9875}{1.0125}} = 49.38\,\Omega$$

$$Z_{0e}^2 = 50\sqrt{\frac{1.125}{0.875}} = 56.69\,\Omega$$

$$Z_{0o}^2 = 50\sqrt{\frac{0.875}{1.125}} = 44.10\,\Omega$$

Simulare



11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:51:07
e:\serenade\PX7_8\PX7_8.ckt
0.00
-25.00
-50.00
-75.00
-100.00
-125.00
Y 1
Through
Coupling
Reflection
Isolation
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

The Lange Coupler

- allows achieving coupling factors of 3 or 6 dB

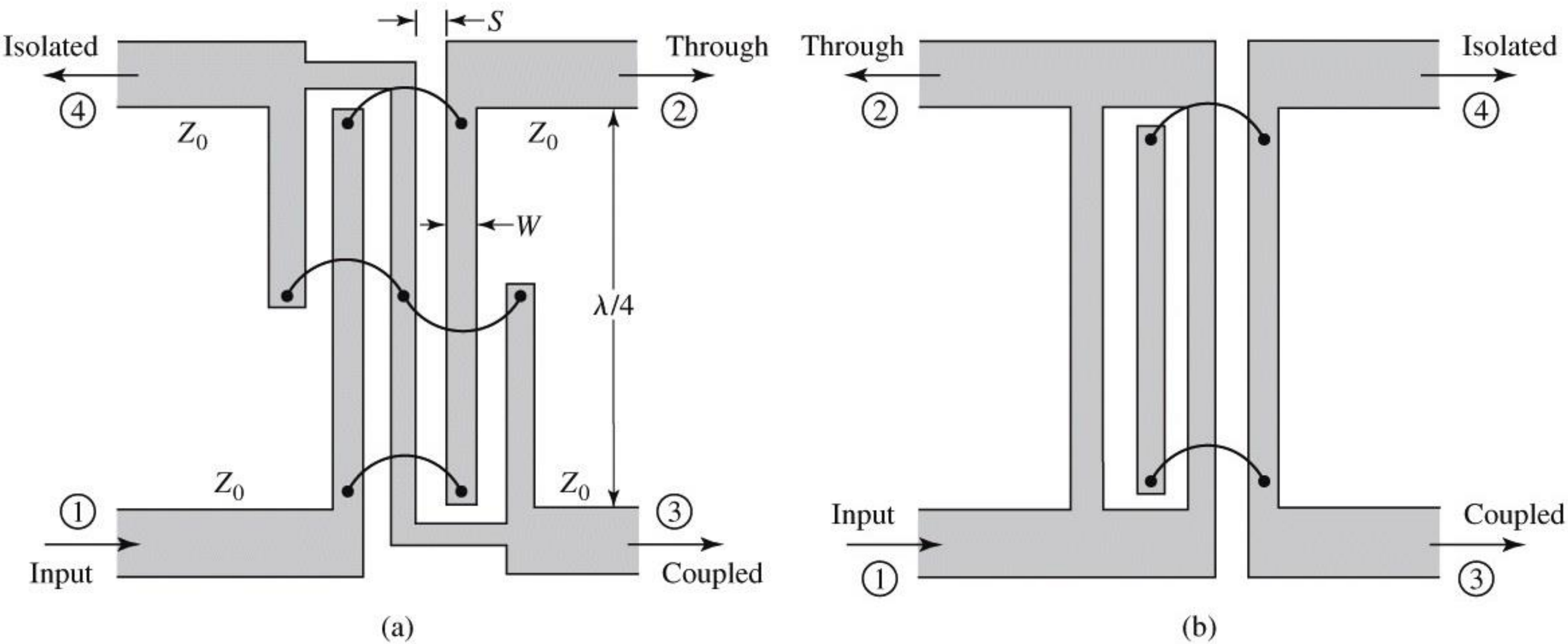


Figure 7.38

The Lange Coupler

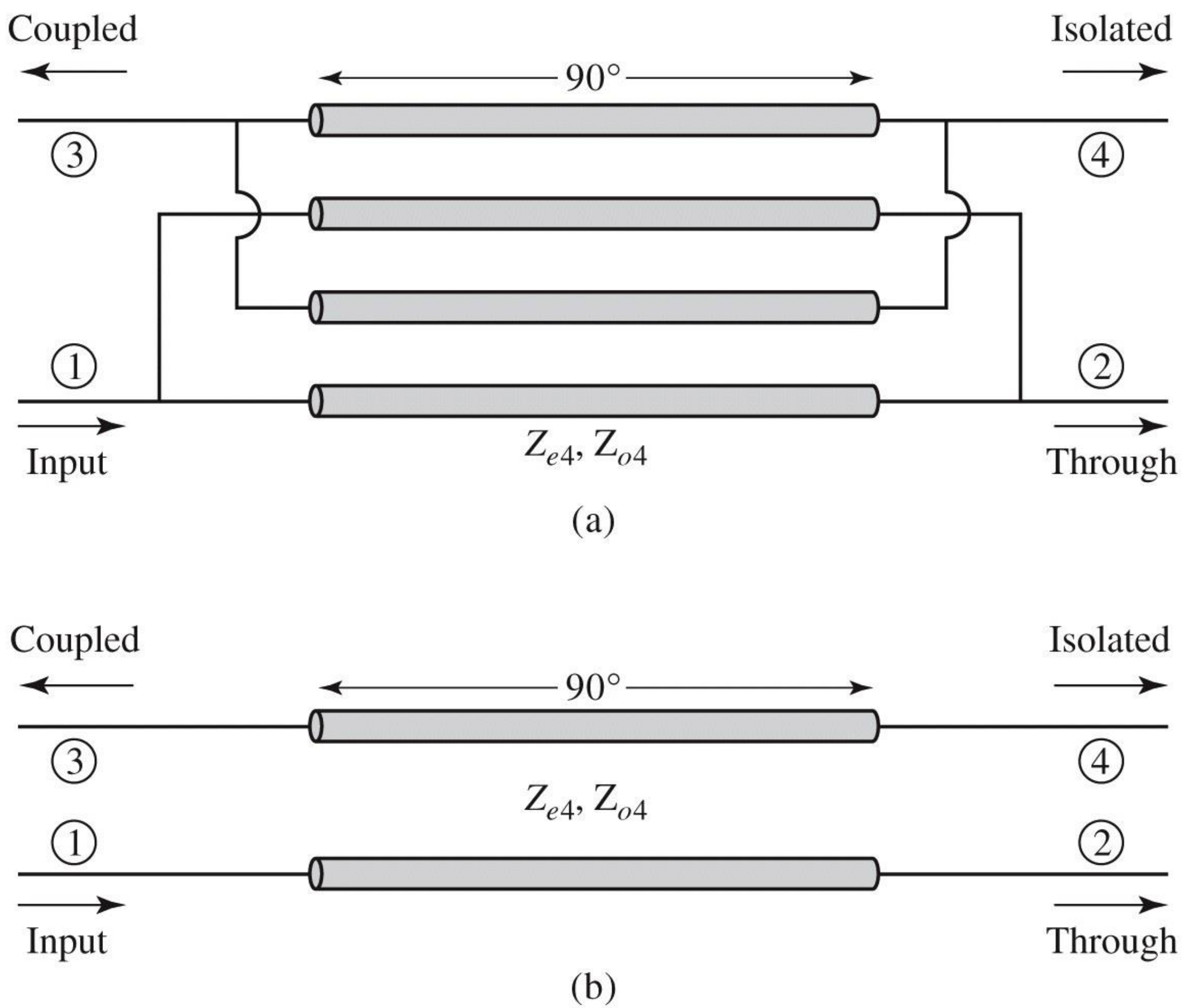
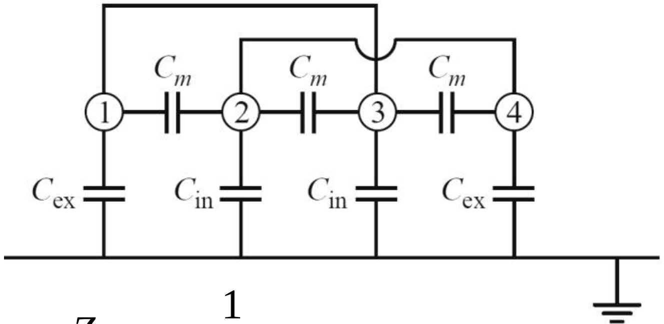


Figure 7.39

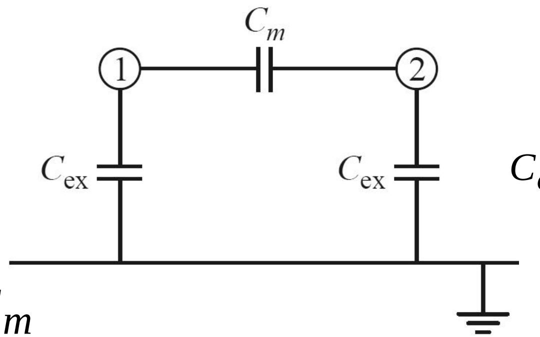
Circuit model



$$C_{in} = C_{ex} - \frac{C_{ex}C_m}{C_{ex} + C_m}$$

$$C_{e4} = C_{ex} + C_{in}$$

$$C_{o4} = C_{ex} + C_{in} + 6C_m$$



$$C_e = C_{ex}$$

$$C_o = C_{ex} + 2C_m$$

$$Z_{e4} = \frac{1}{vC_{e4}}$$

$$Z_{o4} = \frac{1}{vC_{o4}}$$

$$Z_0 = \sqrt{Z_{e4}Z_{o4}} = \sqrt{\frac{Z_{0e}Z_{0o}\left(Z_{0o} + Z_{0e}\right)^2}{\left(3Z_{0o} + Z_{0e}\right)\left(3Z_{0e} + Z_{0o}\right)}}$$

$$C_{e4} = \frac{C_e\left(3C_e + C_o\right)}{C_e + C_o}$$

$$Z_{e4} = Z_{0e}\frac{Z_{0e} + Z_{0o}}{3Z_{0o} + Z_{0e}}$$

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} = \frac{3\left(Z_{0e}^2 - Z_{0o}^2\right)}{3\left(Z_{0e}^2 + Z_{0o}^2\right) + 2Z_{0e}Z_{0o}}$$

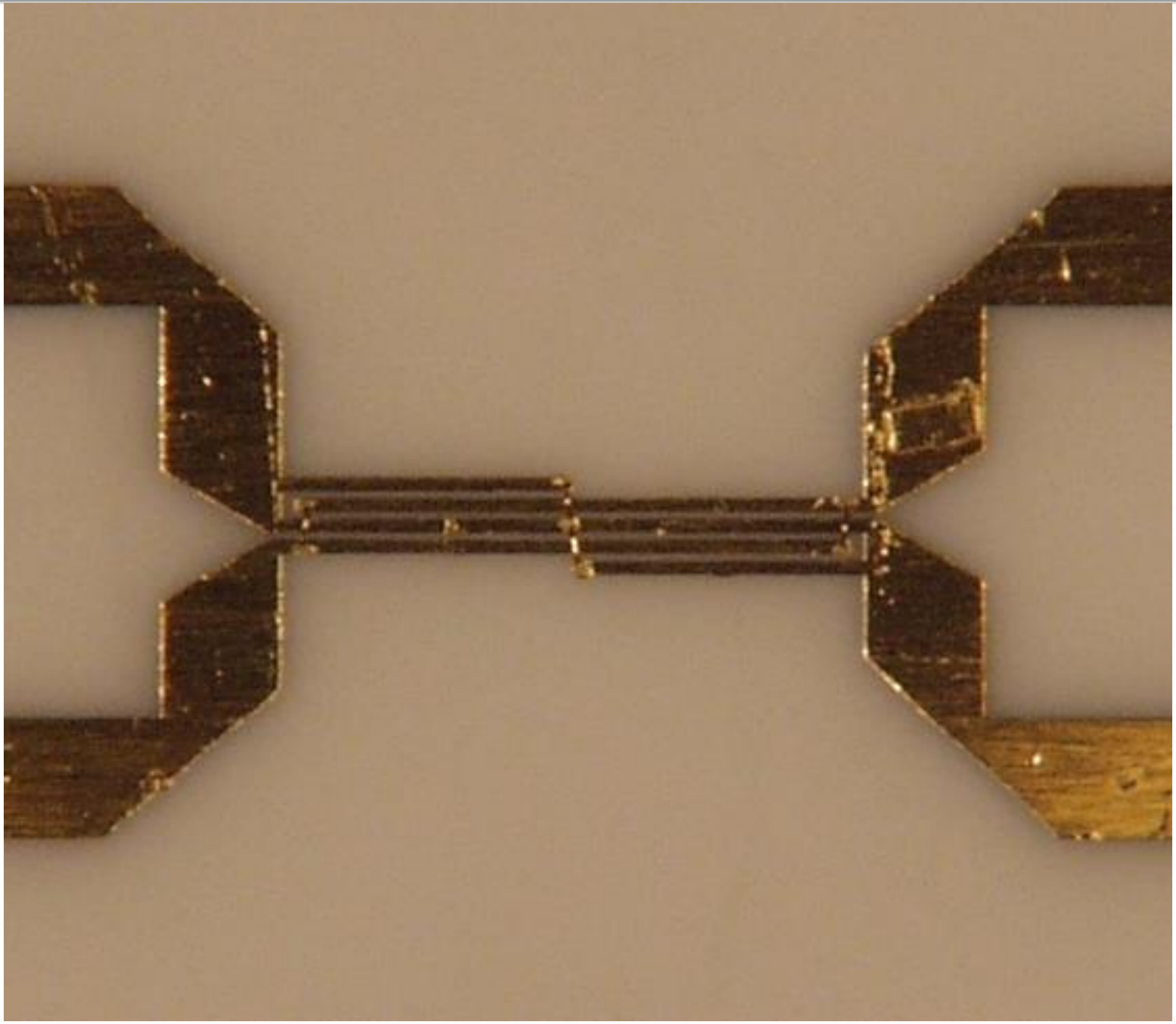
$$C_{o4} = \frac{C_o\left(3C_o + C_e\right)}{C_e + C_o}$$

$$Z_{o4} = Z_{0o}\frac{Z_{0e} + Z_{0o}}{3Z_{0e} + Z_{0o}}$$

$$Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1 - C)/(1 + C)}}Z_0$$

$$Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1 + C)/(1 - C)}}Z_0$$

The Lange Coupler



Directional Couplers

Laboratory no. 2

Directional Coupler

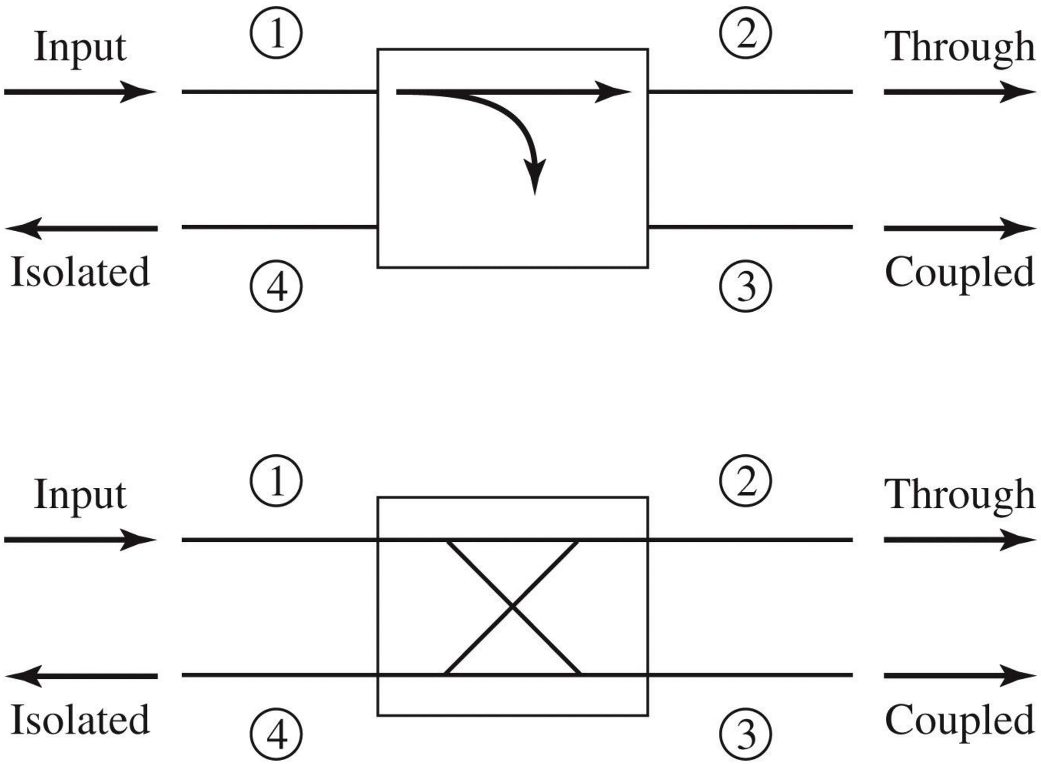


Figure 7.4

$$\left|S_{12}\right|^2 = \alpha^2 = 1-\beta^2$$

$$\left|S_{13}\right|^2 = \beta^2$$

Cuplaj

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivitate

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{\left|S_{14}\right|}\right)[\mathrm{dB}]$$

Izolare

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log\left|S_{14}\right|\,[\mathrm{dB}]$$

$$I = D + C\ ,\ \ dB$$

The cuadrature (90°) hybrid

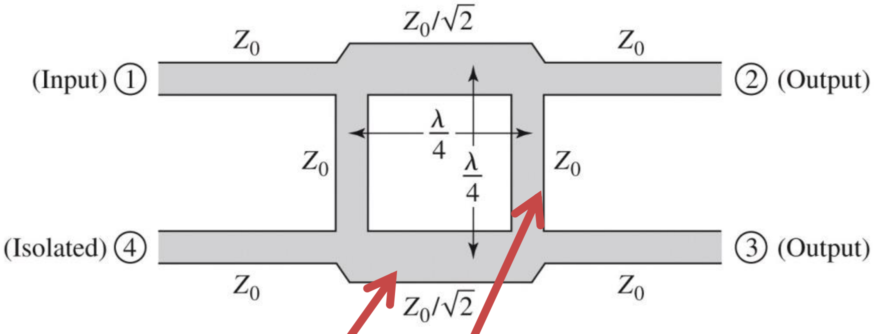


Figure 7.21

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\text{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

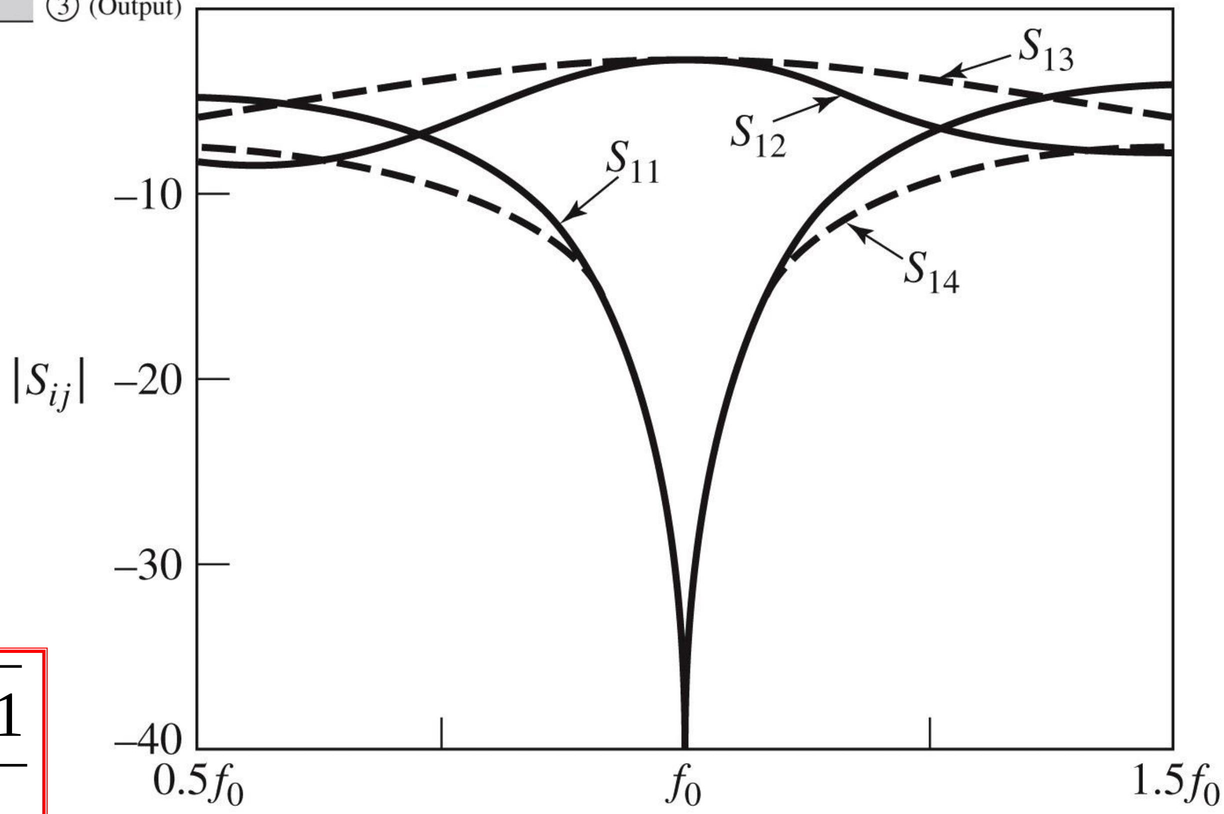
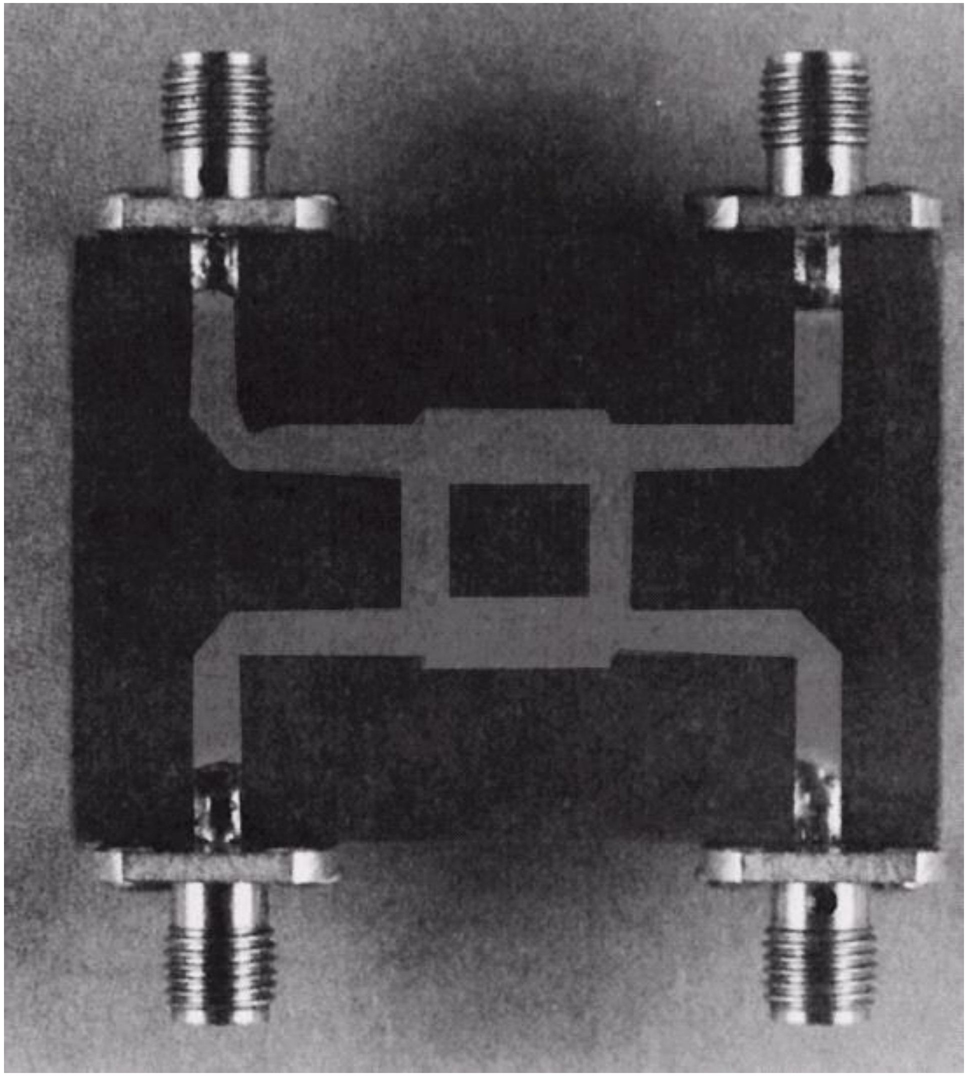
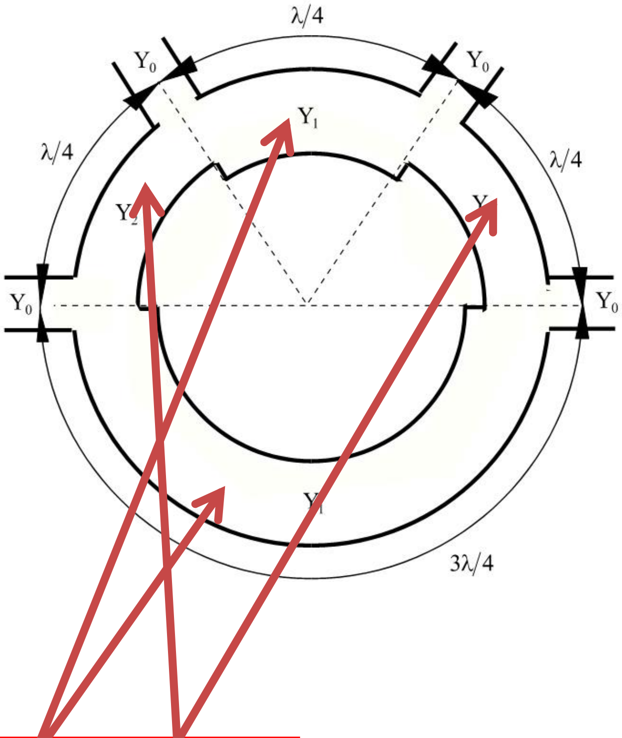


Figure 7.25

Quadrature coupler



The 180° ring hybrid (rat-race)



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\text{dB}] = -20 \cdot \log_{10}(y_1)$$

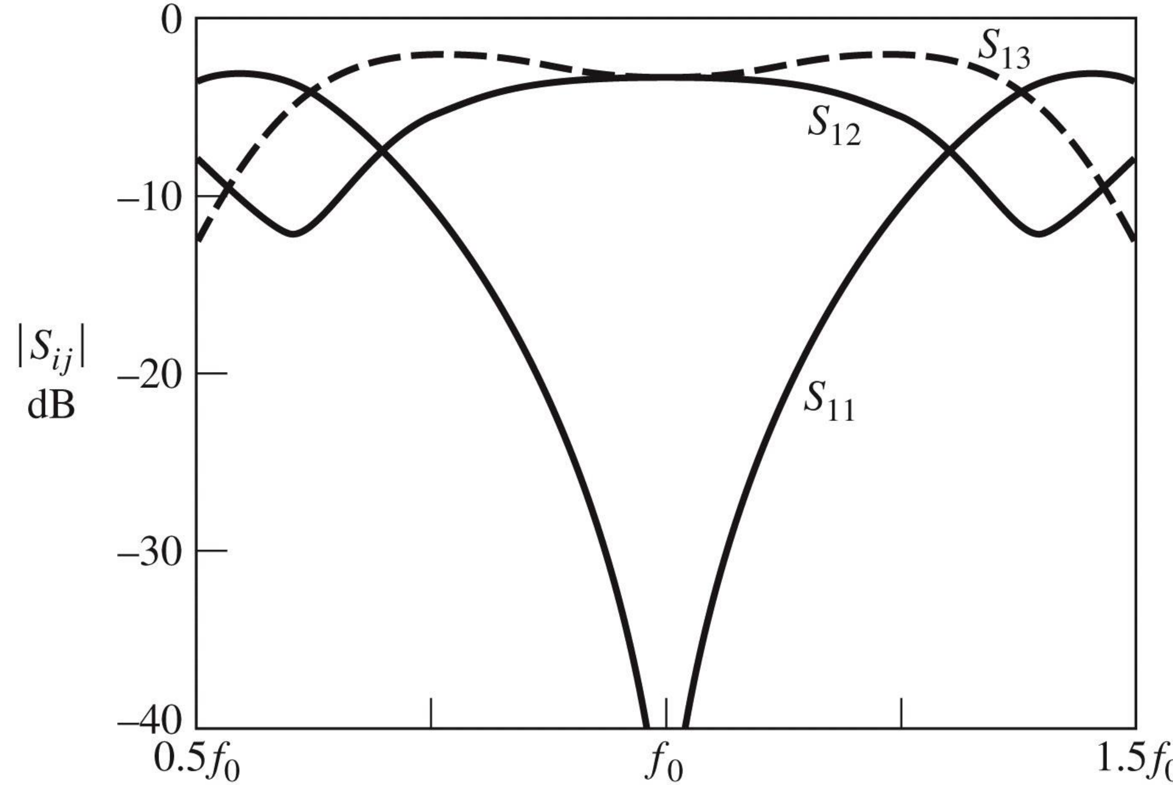


Figure 7.46

Ring coupler

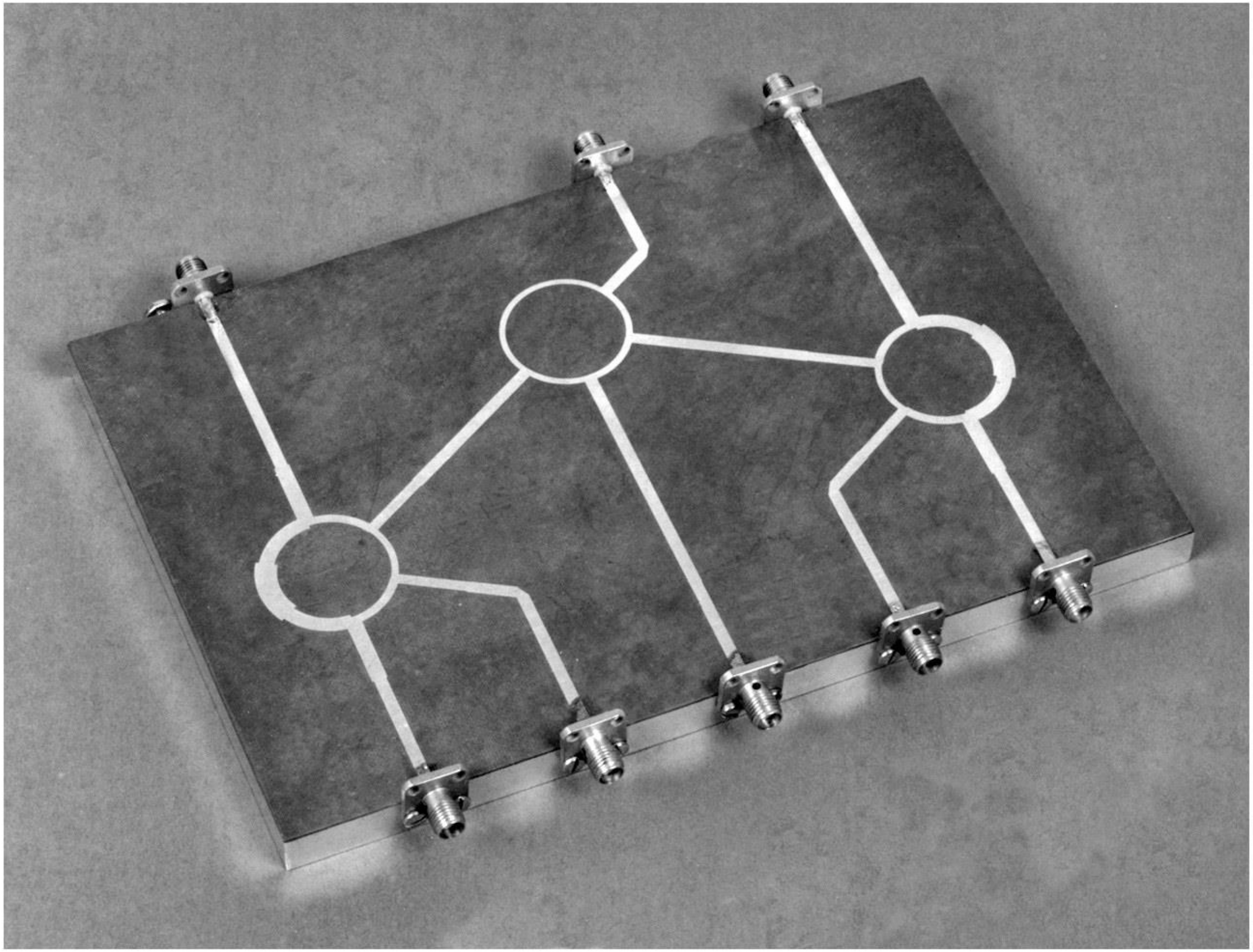
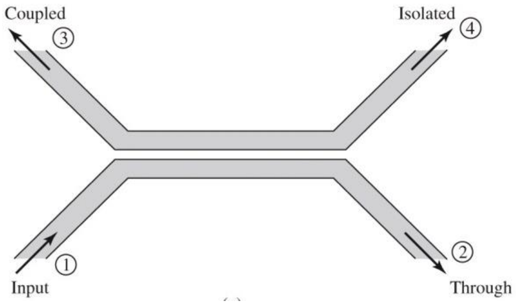


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler



$$Z_{ce} Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

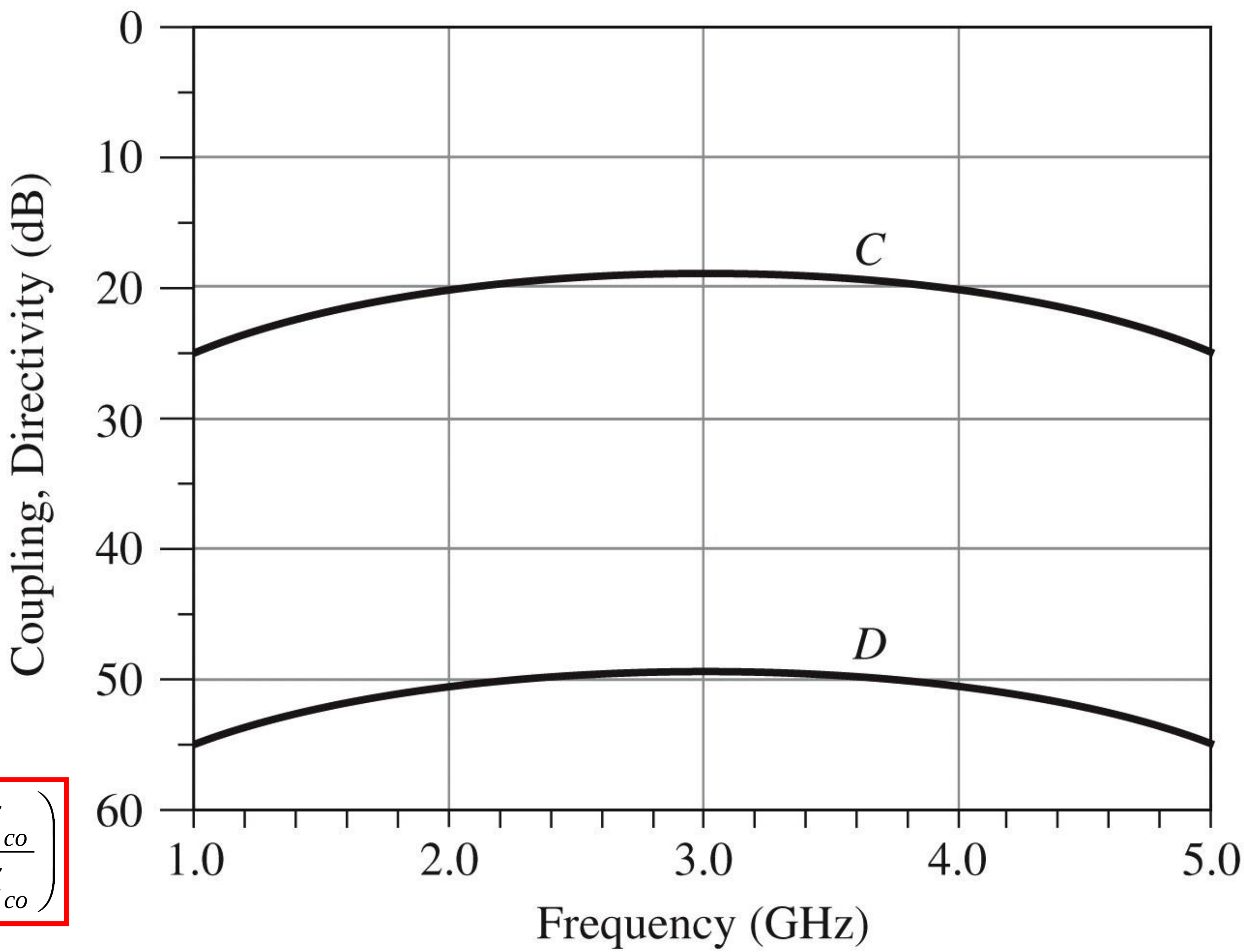
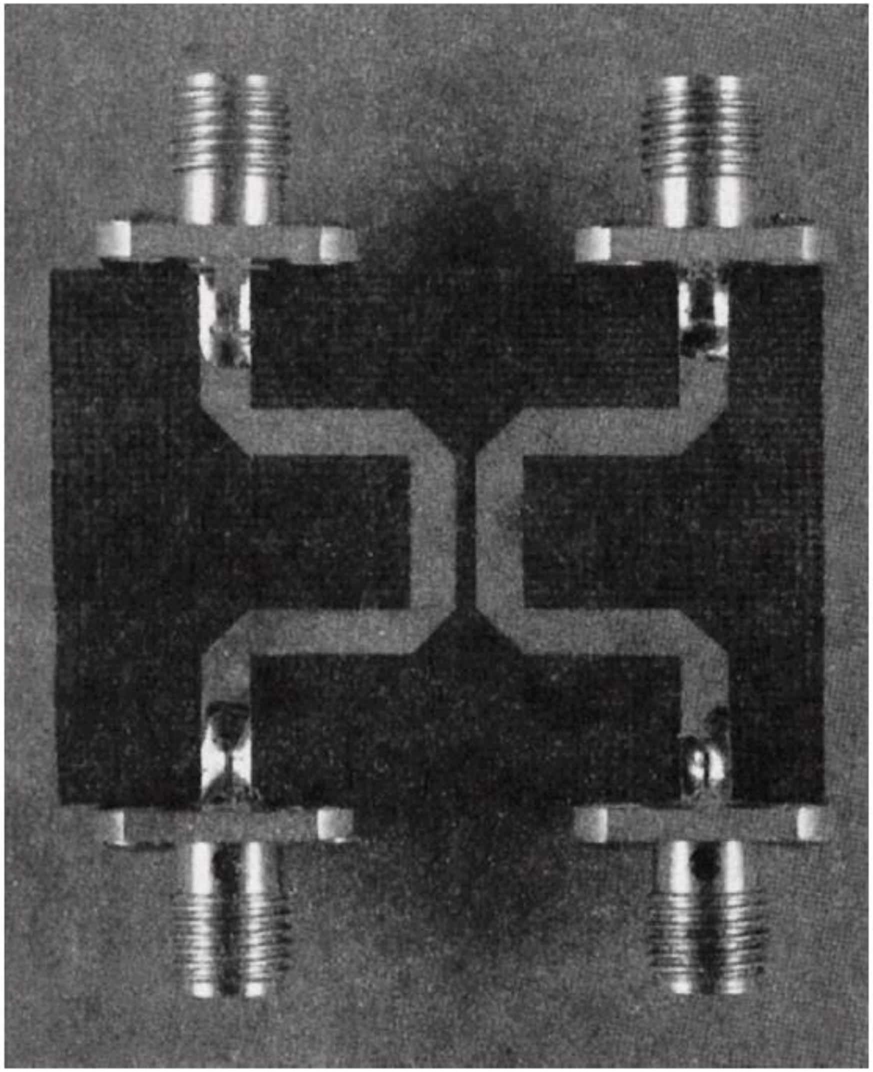


Figure 7.34

Coupled line coupler



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